

**IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
SHERMAN DIVISION**

THE STATE OF TEXAS, et al.,

Plaintiffs,

v.

GOOGLE LLC,

Defendant.

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Civil Action No. 4:20-cv-00957-SDJ

DECLARATION OF ROBERT J. McCALLUM

I, Robert J. McCallum, pursuant to 28 U.S.C. § 1746, hereby declare as follows:

1. I am Of Counsel at Freshfields Bruckhaus Deringer US LLP, which represents Defendant Google LLC (“Google”) in the above-captioned case.

2. I am an attorney admitted to practice in the State of New York and in the United States District Courts for the Southern District of New York and the Eastern District of New York. I have been admitted to appear *pro hac vice* in the above-captioned case.

3. I respectfully submit this Declaration in support of Google’s Motion to Dismiss Plaintiffs’ Fourth Amended Complaint (“FAC”), dated February 8, 2024.

4. Attached as Exhibit A to this Declaration is a true and correct copy of the March 5, 2015 Digiday news article titled “Google sweetens deal for publishers with dynamic price floors” referenced by Plaintiffs. FAC ¶ 535.

5. Attached as Exhibit B to this Declaration is a true and correct copy of a May 12, 2016 Google Blog Post titled “Smarter Optimizations to Support a Healthier Programmatic Market” referenced by Plaintiffs. FAC ¶ 536.

6. Attached as Exhibit C to this Declaration is a true and correct copy of a March 6, 2019 Google Blog Post titled “Simplifying programmatic: first price auctions for Google Ad Manager” referenced by Plaintiffs. FAC ¶ 587.

7. Attached as Exhibit D to this Declaration is a true and correct copy of the September 2014 article titled “Yield Optimizations of Display Advertising with Ad Exchange” referenced by Plaintiffs. FAC ¶ 551.

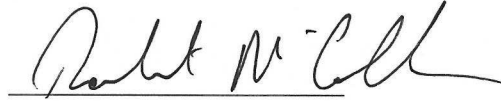
8. Attached as Exhibit E to this Declaration is a true and correct copy of a Google Privacy Policy from December 19, 2019 referenced by Plaintiffs. FAC ¶¶ 573, 574, 579.

9. Attached as Exhibit F to this Declaration is a true and correct copy of a Google Blog Post from April 2020 titled “Learn the basics - How Open Bidding Works” referenced by Plaintiffs. FAC ¶ 587.

10. Attached as Exhibit G to this Declaration is a true and correct copy of a certified translation of *Aguadilla Paint Ctr., Inc. v. Standard Oil*, 2011 TSPR 194, 183 P.R. Dec. 901, 932-33 (2011).

11. I declare under penalty of perjury that the foregoing is true and correct.

Executed on the 8th day of February 2024, in New York, New York.

A handwritten signature in black ink, appearing to read "Robert J. McCallum", written over a horizontal line.

Robert J. McCallum

EXHIBIT A



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Google sweetens deal for publishers with dynamic price floors

March 5, 2015 • 4 min read • By [Digiday](#).



Google is testing a new way for publishers to maximize their advertising revenue.

The feature — called a “dynamic price floor” by a publishing executive briefed on the plans — would automatically change the minimum sell price for ads sold through Google’s DoubleClick Ad Exchange (AdX) based upon advertiser demand.

A price floor is the lowest price at which an ad can be sold for through an ad exchange (a marketplace in which advertisers and publishers buy and sell ad inventory in real time). Publishers set price floors to ensure their ad inventory fetches at least a certain amount of money in a real-time auction. If a publisher sets a price floor of \$2, it will only accept advertiser bids of \$2 or more.

Dynamic price floors, however, change in real time in order to reflect the bid history for a given piece of inventory and ensure that it is sold at a price that better reflects its market rate. For example, say a publisher routinely sells a given ad through an exchange at \$5 CPM, but the ad has a price floor of \$3. Now, say an advertiser that often buys that ad at \$5 decides to bid the minimum of \$3. With a regular price floor, the ad would be sold at \$3. But a dynamic price floor would take an advertiser’s bid history into consideration, pushing the minimum sell price up to \$4 without the publisher having to change its settings.

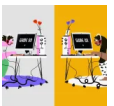
In this sense, automating the price floor entry process helps publishers as it alleviates concerns that AdX will depreciate their ad inventory. Although not as widely prevalent now, publishers have long worried that ad exchanges — and programmatic ad buying, in general — would exert downward pressure on the price of their advertising.

But Google’s dynamic price floor feature comes with a caveat, according to industry executives with knowledge of

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HARIBO, Twitch, Regeneron and WSJ's The Trust are among this year's Digiday Content Marketing Awards finalists



the feature: Certain settings within the feature will only be available to publishers that use Google's ad server and ad exchange together. And that raises issues of whether Google is bundling the two products together so as to further its dominance in the ad tech industry.

Google's dynamic price floor feature will have three priority settings – “low,” “medium” and “high” – that will allow publishers to determine how aggressively they price their inventory, according to those with knowledge of the feature. The high setting would push the minimum sell price higher than the medium setting would have, for instance.

Google has told publishers that the high and medium settings would only be available to those that use its exchange and ad server in conjunction, the executives told Digiday.

“I would have grave concerns if the largest and arguably most important digital intermediary improperly used its market dominance to gain other deals,” Jason Kint, CEO of digital publishing trade group Digital Content Next.

Google disputed that any parts of its product would be exclusive to publishers that use both its ad server and ad exchange.

“That description doesn't match anything in our current product suite or future roadmap. We have and are working on a number of publisher tools to help maximize revenues,” spokeswoman Andrea Faville said in a statement. “For example, in DoubleClick for Publishers, we've had our dynamic allocation feature since 2010, integrated with the DoubleClick Ad Exchange. Separately, we're experimenting with new ways to improve yield for our publishers on the DoubleClick Ad Exchange, but nothing new has launched.”

According to the executives briefed by Google, the dynamic price floor feature would function similarly to Google's “enhanced dynamic allocation,” an ad tech feature that caused Google competitors to complain in spring 2013

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Commerce media grows more dominant by the year, forcing media agencies to keep pace with change



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HARIBO, Twitch, Regeneron and WSJ's The Trust are



to the Federal Trade Commission that Google was abusing its market power.

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With enhanced dynamic allocation, a publisher's ad server selects the highest bid for an ad, even if the publisher had previously sold that ad impression directly to an advertiser. But enhanced dynamic allocation only works with bids that come in through Google's ad exchange instead of selecting the highest bid from across a variety of ad exchanges and ad optimization technologies, according to [a Google customer support page](#).

That Google was bundling its ad server, which was dominant in the industry, with its ad exchange was enough to elicit an FTC probe, according to [a May 2013 Wall Street Journal report](#). The [FTC raised similar questions again last year](#), but it's unclear if the two examinations were connected.

Using DFP and AdX together does offer the distinct advantage of simplifying the increasingly complex world of programmatic advertising.

"There are advantages to having them coupled, especially with programmatic," Kavata Mbondo, head of programmatic solutions at Time Inc., which used both AdX and DFP. "It would be great to have an aggregator of the demand aggregators. That would be awesome. ... But companies don't work like that."

Image courtesy [Getty Images](#)



<https://digiday.com/?p=108581>



Stories like this, in your
inbox each morning

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Job Title

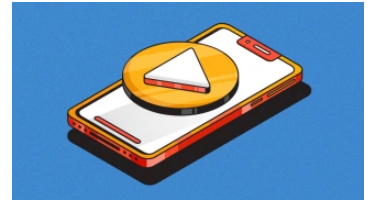
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Digiday+ Research: Nielsen gets boost in NewFronts, upfront cycle — but Comscore wins as measurement provider

Digiday's survey found that Comscore and Nielsen will be the dominant measurement providers during this year's NewFronts and upfront cycle. More than half of publishers said they will accommodate Comscore this year, and nearly half said they will accommodate Nielsen.



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Gannett's Q1 earnings report boasts optimism, despite ad declines and stagnant subscriptions

Gannett's CEO Mike Reed is confident that tides will turn for the company as soon as Q2.



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Despite agencies' investments in data tech, advertiser expectations still fall short

The most insightful customer data sits at the heart of commerce media — but that doesn't mean advertisers always get the insights they're looking for.



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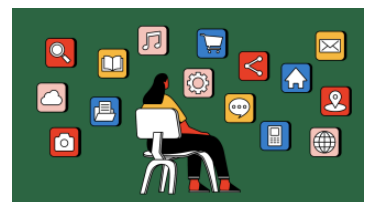
Sponsored by Wurl Streaming is having its moment in the spotlight. Last year, streaming surpassed cable viewing for the first time. Meanwhile, the number of adults accessing streaming video platforms has outpaced those with cable or satellite service. With audiences continuing to flock to Internet-enabled viewing, the opportunity for streamers and content publishers has never [...]



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TikTok, Meta, Vevo announce new ad products and planning tools on NewFronts' final day

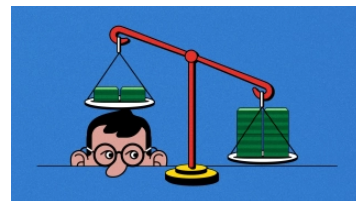
TikTok, Meta and Vevo announced new ad products and media planning tools, and Condé Nast highlighted its popular live events on the final day of the IAB NewFronts.



MEMBER EXCLUSIVE

Media Briefing: Ad spending is slowing, but it's a return to 'normal'

Publishers' ad revenue was still up on an average of 26% year over year in 2022, but outlook for 2023 more so resembles pre-pandemic realities.



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EXHIBIT B



GOOGLE AD MANAGER

Smarter optimizations to support a healthier programmatic market

May 12, 2016 · 2 min read

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Jonathan Bellack
Director, Product Management

Our goal has always been to help publishers and advertisers thrive and create sustainable businesses. For many years Google has used optimization and machine learning techniques to improve the performance of our ads products, and now we're happy to share that we've been extending those techniques to DoubleClick Ad Exchange customers. Today we are introducing Optimized Private Auctions and optimized pricing in the Open Auction to help our publisher partners grow their revenue and give programmatic buyers greater access to premium inventory.

More control of Private Auctions

Private Auctions were developed to help publishers negotiate higher prices by creating special segments of inventory for preferred buyers. As deal volume has grown, we discovered an additional opportunity for publishers to make even more money with Private Auctions. On average, 5% of Private Auction impressions on our platform have an Open Auction buyer willing to pay more than the Private Auction deal price. If all of these bids from Open Auction buyers were able to win their auctions, publishers would see a significant lift to their programmatic revenue.

Optimized Private Auctions, now available to all publishers using DoubleClick Ad Exchange globally, give publishers the ability to allow high-value Open Auction bids to compete against Private Auctions. Full transparency is available to buyers, who can see in the DoubleClick Ad Exchange UI which of their Private Auctions are being optimized.

Greater accuracy with optimized pricing in the Open Auction

In addition to helping publishers maximize revenue from Private Auctions, we've been experimenting with optimized pricing to help publishers set price floors in the Open Auction that more closely reflect the value of their inventory.

The Open Auction tends to have a large price gap between what a buyer bids and what they pay. We've observed more than a 50%¹ price gap between bid and closing prices in many cases. Publishers see this gap as a revenue opportunity and try to close the gap by applying manually-calculated price floors. This is difficult to do well and can lead to lost revenue, or to complex implementations such as offering the same query repeatedly at different price floors that can increase user latency and hurt advertiser performance. We think there's a better way.

Optimized pricing in the Open Auction uses historical data to automate the post-auction analysis and updating of floor prices that publishers already do, and takes it a step further. Not only does our technology use signals like ad unit and device, it also calculates audience-based floors, so publishers can fully benefit from building valuable audiences. And as we've always done, if there is a floor applied to an impression, whether publisher or algorithmically set, we share it with buyers in our bid requests.

In our experiments to date, we have applied optimized pricing to about 15% of transactions, creating over 5% lift in revenue for publishers using the Open Auction. As we expand our experiments with optimized pricing, we will monitor its performance to ensure advertisers continue to get great ROI.

Increasing price transparency

While Optimized Private Auctions and optimized pricing in the Open Auction help publishers get more value for their inventory, they raise important questions. In our conversations, programmatic buyers and sellers have expressed a strong desire for greater transparency and openness in how advertising is valued and prices are set. As the programmatic ecosystem continues to grow, we look forward to partnering with buyers and sellers in an open discussion on price transparency in the industry.

¹ Google internal data, desktop and mobile web impressions in North America



EXHIBIT C



GOOGLE AD MANAGER

Simplifying programmatic: first price auctions for Google Ad Manager

Mar 06, 2019 · 2 min read

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**Sam Cox**

Group Product Manager, Google Ad Manager

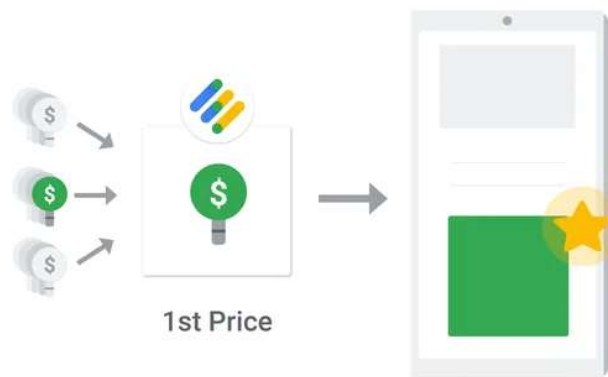
In the very early days of programmatic buying, publishers typically used only one auction to sell their ad inventory. Today the programmatic ecosystem has evolved into a much more complex marketplace where a single ad can pass through a mix of auctions, with different rules, before a winning bid price is selected and an ad is served. This complexity has made it difficult for advertisers and agencies to properly value programmatic inventory and it has driven our publishers and app developers to implement increasingly complicated ad monetization strategies, reducing transparency across the industry. Further, the increasing intricacy of programmatic has made it operationally very difficult, even for experts, to determine what's going well and what needs to be improved.

In order to help simplify programmatic for our partners, in the coming months we'll start to transition publisher inventory to a unified first price auction for Google Ad Manager. We expect the transition to be complete by the end of this year. By switching to a single first price auction, we can help reduce complexity and create a fair and transparent market for everyone.

Common auction scenario today



Unified first price auction



What this means for our partners

With this change, every offer from programmatic buyers will compete in the same unified auction, alongside inventory which is directly negotiated with advertisers. An advertising buyer's bid will not be shared with another buyer before the auction or be able to set the price for another buyer. The buyer that wins the auction pays the price they bid. By simplifying our auction in Ad Manager, we can help make it easier for publishers and app developers to manage and get fair value for their inventory.

Preparing for this change

Since the change from second to first price will require both buyers and sellers to make changes in their programmatic strategies, we'll give everyone time to prepare over the next few months before we start testing. During this time, publishers and app developers will need to rethink how they use price floors and technology partners will need to adjust how they bid for Google Ad Manager Inventory.

It's important to note that our move to a single unified first price auction only impacts display and video inventory sold via Ad Manager. This change will have no impact on auctions for ads on Google Search,

AdSense for Search, YouTube, and other Google properties, and advertisers using Google Ads or Display & Video 360 do not need to take any action.

We are excited to take this next step to simplify the programmatic ecosystem and help our partners grow. As we get closer to the start of our transition to a unified first price auction, we'll work with all our partners to help them get prepared. ■

POSTED IN:

Google Ad Manager

EXHIBIT D



Yield Optimization of Display Advertising with Ad Exchange

Author(s): Santiago R. Balseiro, Jon Feldman, Vahab Mirrokni and S. Muthukrishnan

Source: *Management Science*, December 2014, Vol. 60, No. 12 (December 2014), pp. 2886–2907

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Yield Optimization of Display Advertising with Ad Exchange

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It is clear from the growing role of ad exchanges in the real-time sale of advertising slots that Web publishers are considering a new alternative to their more traditional reservation-based ad contracts. To make this choice, the publisher must trade off, in real-time, the short-term revenue from ad exchange with the long-term benefits of delivering good spots to the reservation ads. In this paper we formalize this combined optimization problem as a multiobjective stochastic control problem and derive an efficient policy for online ad allocation in settings with general joint distribution over placement quality and exchange prices. We prove the asymptotic optimality of this policy in terms of any arbitrary trade-off between the quality of delivered reservation ads and revenue from the exchange, and we show that our policy approximates any Pareto-optimal point on the quality-versus-revenue curve. Experimental results on data derived from real publisher inventory confirm that there are significant benefits for publishers if they jointly optimize over both channels.

Data, as supplemental material, are available at <http://dx.doi.org/10.1287/mnsc.2014.2017>.

Keywords: dynamic programming-optimal control; Internet advertising; revenue management

History: Received August 26, 2011; accepted May 26, 2014, by Dimitris Bertsimas, optimization. Published online in *Articles in Advance* October 17, 2014.

1. Introduction

“Internet display advertising” refers generally to the graphical and video ads that are now ubiquitous on the Web. These types of ads generated approximately 11 billion dollars in the United States in 2012, and analysts see a clear rising trend (Internet Advertising Bureau 2013). Traditionally, an advertiser would buy display ad placements by negotiating deals directly with a publisher (the owner of the webpage) and signing an agreement, called a guaranteed contract. These deals usually take the form of a specific number of ad impressions reserved over a particular time horizon (e.g., one million impressions over a month). A publisher can make many such deals with different advertisers, with potentially sophisticated relationships between the advertisers’ targeting criteria. The publisher would then need to assign arriving impressions to the matching reservations so as to maximize the *placement quality* of the contracts. Typically, the probability that a user clicks on an ad (known as *click-through rate*) is used as a metric of placement quality.

Guaranteed contracts can suffer in efficiency: Since slots are booked in advance, both parties cannot react to instantaneous changes in traffic patterns or market conditions. However, this has changed in the last

couple of years. Advertisers may now purchase ad placements through spot markets for the real-time sale of online ad slots, called ad exchanges. Prominent examples of exchanges are Yahoo!’s RightMedia, Microsoft’s AdECN, and Google’s DoubleClick and OpenX. Although exchanges differ in their implementations, in a generic ad exchange (AdX), publishers post an ad slot with a reservation price, advertisers post bids, and an auction is run; this happens between the time a user visits a page and the ad is displayed (Muthukrishnan 2009). Ad exchanges allow advertisers to bid in real time and pay only for valuable customers, instead of bulk buying impressions and targeting large audiences.

In the presence of AdX, publishers face the multiobjective problem of maximizing the overall placement quality of the impressions assigned to the reservations together with the total revenue obtained from AdX, while complying with contractual obligations. These two objectives are potentially conflicting; in the short term, the publisher might boost the revenue stream from AdX at the expense of assigning lower-quality impressions to the advertisers. In the long term, however, it may be convenient for the publisher to prioritize the advertisers in view of attracting future contracts. So for a given piece of ad inventory, the publisher must quickly decide whether to send the inventory to AdX

(and at what price), or to assign it to an advertiser with a reservation. In this paper, we study the problem faced by the publisher, jointly optimizing over AdX and the reservations.

1.1. Main Contributions

The contributions of this paper are as follows:

First, we bring to bear techniques of revenue management (RM) and model the publisher’s problem as a combination of a *capacity allocation problem* to handle the guaranteed contracts together with a *dynamic pricing problem* to handle the reserve price optimization in the ad exchange. We tackle the publisher’s multiobjective problem by taking a weighted sum of (i) the revenue from AdX and (ii) the placement quality of the contracts, and we show how to construct the Pareto-efficient frontier of attainable objectives. Using data derived from real publisher inventory, we show empirically that the Pareto-efficient frontier is highly concave and that there are significant benefits for publishers if they jointly optimize over both channels.

The publisher’s problem can be thought of as a parallel-flight network RM problem (see, e.g., Talluri and van Ryzin 1998) in which users’ click probabilities are requests for itineraries, and advertisers are edges in the network. As in the prototypical RM problem, we look for a policy maximizing the ex ante expected revenue, which can be obtained by using dynamic programming (DP). There are three differences, however, with the traditional network RM problem. First, we aim to satisfy all contracts, or completely deplete all resources by the end of the horizon. Second, in the traditional problem, requests are for only one itinerary (which can be accepted or rejected), whereas in our model each impression can be potentially assigned to any contract and the publisher needs to decide to whom to assign the impression based on possibly correlated placement qualities. Finally, publishers in display advertising may submit impressions to a spot market to increase their revenues, which adds a dynamic pricing dimension to the problem. To this end we introduce a general model of targeting based on the user’s attributes that takes into account the potential correlation between guaranteed contracts’ placement quality and exchange’s bids. Table 1 summarizes these points.

Second, because of the so-called “curse of dimensionality,” the optimal policy cannot be computed efficiently in most real-world problems; instead we aim for a deterministic approximation in which stochastic quantities are replaced by their expected values and quantities are assumed to be continuous (Gallego and van Ryzin 1994). As a result, we derive a provably good stationary policy that resembles a bid-price control that is extended with a pricing function to account for AdX (Theorem 1). Our policy assigns each

Table 1 Comparison of the Display Ad and Network Revenue Management Problems

	Network RM	Display Ad
Resources	Seats	Impressions
Edges	Flight legs	Contracts
Capacity constraints	\leq	$=$
Objective	Maximize revenue	Maximize placement quality
Decision	Accept/reject itinerary request	Determine best matching contract
Spot market/dynamic pricing	No	Yes

guaranteed contract a bid price (or dual variable), which may be interpreted as the opportunity cost of assigning one additional impression to the reservation. When a user arrives, the pricing function quotes a reserve price to submit to the exchange that depends on the opportunity cost of assigning the impression to an advertiser (and potentially on the impression’s attributes). If no AdX bid exceeds this reserve price, the impression is immediately assigned to the advertiser whose placement quality exceeds its bid price by the largest amount. A salient feature of our policy from the managerial standpoint is its simplicity: the publisher only needs to keep track of a single pricing function for the exchange, and one bid price for each contract that is obtained, in turn, by solving a convex stochastic minimization problem.

The optimal policy always tests the exchange before assigning an impression to a guaranteed contract because the loss of not assigning an impression of high quality to the reservation can be compensated by choosing an adequately high reserve price. This result implicitly hinges upon (i) the absence of a fixed cost for accessing the exchange and (ii) the publisher’s ability to dynamically adjust the reserve price for each impression based on user attributes. In the presence of a fixed cost, the publisher tests the exchange only if the contract’s opportunity cost is less than or equal to a fixed threshold; in the case of static pricing, the publisher tests the exchange when the expected revenue from AdX exceeds the contract’s opportunity cost.

Third, we introduce a novel probabilistic tie-breaking rule to handle the possibility of multiple advertisers attaining the maximum bid-price adjusted placement quality. The rule breaks ties by randomizing according to a fixed probability distribution, which is predetermined by solving an assignment problem. Combining ideas from combinatorial optimization and convex analysis, we show that such a tie-breaking rule always exists (Proposition 2). This tie-breaking rule is a novelty from the bid-price perspective and allows one to consider general distributions of placement quality. Numerical experiments confirm that ties are a real concern in practical problems; if the publisher fails to

take them into account, significant losses in yield can be incurred.

Fourth, we provide a rigorous bound on the convergence rate of our policy to the optimal online policy (Theorem 2).¹ Our theoretical analysis captures the multiobjective nature of our problem and shows that any Pareto-optimal policy can be approximated by a simple modified bid-price policy that performs asymptotically close both in terms of AdX revenue and the contract quality. The loss of our policy in terms of quality and revenue relative to the optimal one converges to zero at a rate of $O(N^{-1/2})$ where N is the number of impressions in the horizon. The deterministic approximation is thus suitable when the number of impressions in the horizon is large, which fits well in the context of Internet advertising. From a computational standpoint we provide an efficient and simple method of computing the dual variables that is applicable to large instances with many contracts.

Finally, we numerically compare the performance of our policy with two alternative heuristics that are common in practice. The first heuristic is a “greedy policy” that disregards the opportunity cost of capacity and assigns the impression to the advertiser with maximum placement quality. The second is a “static-price policy” that sets a constant reserve price for the exchange throughout the horizon. Our results on actual publisher data show that these heuristics significantly underperform when compared to the optimal policy. From a managerial perspective, these results stress the importance of pondering the opportunity cost of capacity in performing the assignment to the guaranteed contracts, and of pricing dynamically in the exchange to react to the users’ attributes and the value of the reservations.

1.2. Related Work

Our works draw on three streams of literature, namely, that of display advertising with ad exchange, revenue management, and online allocation. Rather than attempting to exhaustively survey the literature on each area, we focus on the work more closely related to ours.

1.2.1. Display Advertising with Ad Exchange.

There has been recent work on display ad allocation with both contract-based advertisers and spot market advertisers. Ghosh et al. (2009) focus on “fair” representative bidding strategies in which the publisher bids on behalf of the contract-based advertisers competing with

the spot market bidders. This line of work is mainly concerned with computing such fair representative bidding strategies for contract-based advertisers. Chen (2011) considers the case when the publisher runs the exchange, and employing a mechanism design approach he characterizes, through dynamic programming, the optimal dynamic auction for the spot market. In this model, both bids from the spot market and the total number of impressions are stochastic. We focus, instead, on combined yield optimization and present a model and an algorithm taking into account any trade-off between quality delivered to reservation ads and revenue from the spot market. Yang et al. (2012) studied the problem faced by the publisher of allocating between the two markets using multiobjective programming. As in our work, they consider different objectives for the publisher, such as minimizing the penalty of underdelivery and maximizing the revenue from the spot market and the representativeness of the allocation. However, they employ a deterministic model with no uncertainty in which future inventory and contracts are nodes in a bipartite graph. Alaei et al. (2009) proposed a utility model that accounts for two types of advertisers: one oriented toward campaigns and seeking to create brand equity, and the other oriented toward the spot market and seeking to transform impressions to sales. Here impressions are commodities that can be assigned interchangeably to any advertisers. In this setting they look for offline and online algorithms aiming to maximize the utility of their contracts.

1.2.2. Revenue Management. Another stream of relevant work is that of RM. Although RM is typically applied to airlines, car rentals, hotels, and retailing (Talluri and van Ryzin 2004), our problem formulation and analysis is inspired by RM techniques. A popular method for controlling the sale of inventory in revenue management applications is the use of bid-price controls. These were originally introduced by Simpson (1989) and thoroughly analyzed by Talluri and van Ryzin (1998). In this setting, a bid-price control sets a threshold or bid price for each advertiser, which may be interpreted as the opportunity cost of assigning one additional impression to the advertiser. This approach is standard in the context of revenue maximization, e.g., the stochastic knapsack problem by Levi and Radovanovic (2010). From this perspective, our contribution is the inclusion of a spot market, the exchange, as a new sales channel.

There is some literature on display advertising from a revenue management angle that focuses exclusively on guaranteed contracts (see, e.g., Araman and Fridgeirsdottir 2011, Fridgeirsdottir and Najafi 2010, Roels and Fridgeirsdottir 2009, and Turner 2012). These papers, however, do not consider the spot market. In the related area of TV broadcasting, Araman and Popescu (2010)

¹ Typically, ad allocation research employs the optimal offline policy in hindsight as a benchmark. Although in the absence of the spot market the performance of the offline and online policies are asymptotically equivalent (see, e.g., Talluri and van Ryzin 1998), in the presence of the spot market this is no longer the case if we assume that the oracle is aware of bids’ realizations.

study the allocation of advertising space between forward contracts and the spot market when the planner faces supply uncertainty.

Finally, in terms of multiobjective optimization in revenue management, Levin et al. (2008) employ a weighted-sum approach to determine, in a dynamic pricing setting, the Pareto-efficient frontier between revenue and the probability that total revenue falls below a minimum acceptable level. Phillips (2012) uses a similar approach to determine the efficient frontier between any two goals that are linear in load (such as revenue and profits) in a single-leg revenue management problem.

1.2.3. Online Allocation. Our work is closely related to online ad allocation problems, including the *display ads allocation* (DA) problem and the *AdWords* (AW) problem. In both problems, the publisher must assign online impressions to advertisers, optimizing the efficiency or revenue of the allocation, while respecting prespecified contracts.

In the DA problem, advertisers demand a maximum number of eligible impressions, and the publisher must allocate impressions that arrive online to them. Each impression has a potentially different value for every advertiser. The goal of the publisher is to maximize the value of all the assigned impressions. The adversarial online DA problem was considered in Feldman et al. (2009), which showed that the problem cannot be approximated without exploiting *free disposal*; using this property (that advertisers are at worst indifferent to receiving more impressions than required by their contract), a simple greedy algorithm is half-competitive, which is optimal. When the demand of each advertiser is large, a $(1 - 1/e)$ -competitive algorithm exists (Feldman et al. 2009), and it is tight. The stochastic model of the DA problem is more related to our problem. Following a training-based dual algorithm by Devanur and Hayes (2009), training-based $(1 - \epsilon)$ -competitive algorithms have been developed for the DA problem and its generalization to various packing linear programs (Feldman et al. 2010, Vee et al. 2010, Agrawal et al. 2014).

In the AW problem, the publisher allocates impressions resulting from search queries. Here each advertiser has a budget on the total spend instead of a bound on the number of impressions. Other than training-based dual algorithms and primal-dual algorithms that get similar bounds as in the DA problem, online adaptive optimization techniques have been applied to online stochastic ad allocation (Tan and Srikant 2012). Such control-based adaptive algorithms achieve asymptotic optimality following an updating rule inspired by the primal-dual algorithms.

Our work differs from all the above in three main aspects. (i) Instead of using the framework of competitive analysis and comparing the solution with

the optimum solution in hindsight, we compare the performance of our algorithm with the optimal online policy. In contrast to the online ad allocation literature, our work assumes that the decision-maker maintains probabilistic “priors” on the primitives. (ii) None of the above work considers the simultaneous allocation of reservations and AdX. In particular, these papers do not consider the trade-off between the revenue from a spot market based on real-time bidding and the efficiency of reservation-based allocation. (iii) Previous work fails to take into account the possibility of ties between contracts, which can result in significant yield losses. Our policy explicitly handles ties by introducing a novel probabilistic tie-breaking rule. Regarding the last two points, in §EC.5 of the online appendix (available at <http://faculty.fuqua.duke.edu/~srb43/papers/adx-alloc-ms-ec.pdf>), we introduce a prior-free policy that learns in a nonparametric fashion its underlying parameters, and we theoretically analyze the expected performance of this policy by borrowing tools from statistical learning theory.

2. Model

Consider a publisher displaying ads on a webpage. The webpage has a single slot for display ads, and each user is shown at most one impression per page. The publisher has signed contracts with A advertisers under which exactly C_a impressions are to be delivered to advertiser $a \in \mathcal{A}$, where we denote by $\mathcal{A} = \{1, \dots, A\}$ the set of advertisers. Neither overdelivery nor underdelivery is allowed.

Although the number of users visiting a webpage is uncertain, publishers usually have fairly good estimates of the total number of expected users that arrive in a given horizon. In this model time periods correspond to users' arrivals, and we assume that the total number of users is fixed and equal to N (a random number of users can be accommodated in our model by considering dummy arrivals). Time periods are indexed backward in time by $n = N, \dots, 1$. Each user is identified by a vector of attributes $U_n \in \mathcal{U}$, where \mathcal{U} is some finite subset of \mathbb{R}^M . The vector of attributes contains information that is relevant to the advertisers' targeting such as (i) the Web address or URL; (ii) keywords related to the content of the webpage; (iii) the dimension and position of the slot on the page; (iv) user's geographical information (i.e., where the user is located); (v) user's demographics, such as education level, gender, age, and income; (vi) user's device and operating system; and (vii) cookie-based behavioral information, which allows bidders to track the user's past activity on the Web. We assume that the vectors of attributes $\{U_n\}_{n=1, \dots, N}$ are random, independent, and identically distributed.

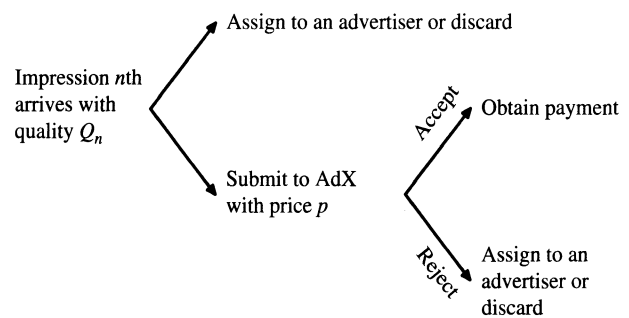
Based on the vector of attributes for the impression U_n , the publisher determines a vector of placement

qualities $Q_n = \{Q_{n,a}\}_{a \in \mathcal{A}}$, where $Q_{n,a}$ is the predicted quality advertiser a would perceive if assigned the impression. Qualities lie in some compact space $\Omega \subseteq \mathbb{R}^A$. A typical measure of placement quality is the estimated probability that the user clicks on each ad. In practice, such a measure of quality is learned by performing, for example, a logistic regression based on the vector attributes as explanatory variables. Here we abstract from the learning problem and assume that the qualities are deterministically determined from the impression attributes (to simplify the notation we omit this dependence). We do allow, however, for qualities to be jointly distributed across advertisers. This captures the fact that advertisers might have similar target criteria, and hence the qualities perceived might be correlated. We do not impose any further restrictions on the qualities, other than bounded support. Note that the publisher observes the realization of the placement quality before showing the ad.

We assume that the number of arriving impressions suffices to satisfy the contracts, or equivalently $\sum_{a \in \mathcal{A}} \rho_a \leq 1$, where $\rho_a = C_a/N$ denotes the capacity-to-impression ratio of an advertiser. An assumption of this general model is that any user can potentially be assigned to any advertiser. In practice each advertiser may be interested in a particular group of user types. It is important to note that this is not a limitation of our results, but rather a modeling choice; in §5 we show how to handle targeting criteria by forcing the publisher to pay a goodwill penalty to the advertisers each time an undesired impression is incorrectly assigned.

Arriving impressions may be assigned to the advertisers, discarded, or auctioned in AdX for profit. In a general AdX (Muthukrishnan 2009), the publisher contacts the exchange to offer a minimum price for the slot. Additionally, the publisher may submit some partial information about the users visiting the website. User information allows bidders in the exchange to target more effectively, which may in turn result in higher bids (see, e.g., Milgrom and Weber 1982). Internally the exchange contacts different ad networks, and in turn they return bids for the slot. The exchange determines the winning bid among those that exceed the reserve price via a second-price auction and returns a payment to the publisher. In this case we say that the impression is *accepted*, and the publisher is contractually obligated to display the winning impression. In the case that no bid attains the reserve price, no payment is made and the impression is *rejected*. We present the formal model of the exchange in §2.2. The entire operation above is executed before the page is rendered on the user's screen. Thus, in the event that the impression is rejected by the exchange, the publisher may still be able to assign it to some advertiser. Figure 1 summarizes the decisions involved.

Figure 1 Publisher's Decision Tree for a New Impression



Note. In the absence of a fixed cost for accessing the exchange, the upper branch is never taken because the publisher is always better off testing AdX with a sufficiently high price that compensates for the yield loss of not assigning the impression.

Note that proofs of selected statements are presented in the main appendix, and the remaining proofs are available in the online appendix. Random variables are denoted by uppercase letters and realizations by their lowercase counterparts. For notational simplicity we extend the set of advertisers to $\mathcal{A}_0 = \{0\} \cup \mathcal{A}$ by including an outside option 0 that represents discarding an impression. We set the quality of the outside option identically to zero; i.e., $Q_{n,0} = 0$ for all impressions $n = 1, \dots, N$. We set $\rho_0 = 1 - \sum_{a \in \mathcal{A}} \rho_a$ to be the fraction of impressions that are not assigned to any advertiser (either accepted by AdX or effectively discarded).

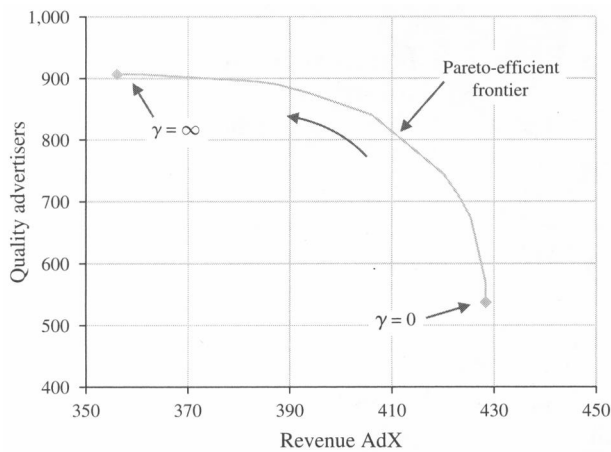
2.1. Objective

The publisher's problem is to maximize the overall placement quality of the impressions assigned to the advertisers together with the total revenue obtained with AdX while complying with contractual obligations. These objectives are conflicting, and there exist potentially many Pareto-optimal solutions. Figure 2 shows the efficient frontier of attainable objectives for a given publisher. We attack the multiobjective problem by taking a weighted sum of both objectives: at the publisher's disposal is a parameter $\gamma \geq 0$; by adjusting this parameter the publisher can construct the convex hull of the Pareto-efficient frontier of attainable revenue from AdX and quality for the advertisers. The aggregated objective is given by

$$yield = revenue(AdX) + \gamma \cdot quality(advertisers).$$

In §6.2 we study experimentally the impact of the choice of γ on both objectives and determine the Pareto frontier for real publisher data.

The parameter γ can be understood as a trade-off between units of quality and units of revenue, and by picking the right value for γ the publisher can balance these conflicting objectives. A large γ (i.e., contracts' placement quality is more valuable than AdX revenue) gives priority to assigning high-quality impressions to the advertisers, whereas a small γ (i.e., contracts'

Figure 2 (Color online) Efficient Frontier of Attainable AdX Revenue and Guaranteed Contract Quality for a Given Publisher

Note. The Pareto-optimal solutions are computed by varying the trade-off parameter γ in the weighted objective.

placement quality is less valuable than AdX revenue) gives priority to the revenue from AdX. The publisher may even set different trade-off parameters for each contract.

When the choice of the trade-off parameter is not clear, the publisher may impose a lower bound on the overall quality of the impressions and then maximize the total revenue from AdX. This may have a more natural interpretation for some publishers and might be simpler than having to set γ . Here the efficient frontier provides the maximum attainable revenue for each target quality and the proper γ to achieve the quality constraint. Alternatively, one can interpret γ as the Lagrange multiplier of the quality of service constraint, and our problem as the Lagrange relaxation of the constrained program. In §EC.1.3 of the online appendix, we analyze the implications of this formulation.

2.2. AdX Model with User Information

For ease of exposition, we assume that there is one bidder in the exchange. In §EC.1.1 of the online appendix, we drop this assumption and show that all results hold under a more general second-price auction with multiple bidders. The publisher submits an impression to AdX accompanied by a minimum acceptable price, denoted by $p \geq 0$. The impression is accepted if there is a bid of value p or more, and the publisher is paid the minimum price p when the impression is accepted. In practice, bids from the exchange may be correlated with the user information disclosed, and publishers maintain different estimates of the bids' distribution as a function of the attributes. We assume that, conditional on the user information $u \in \mathcal{U}$, bids are independent across impressions and identically distributed according to a cumulative distributive function

$F(\cdot; u)$. Hence, when the publisher discloses some information u , the impression is accepted with probability (w.p.) $1 - F(p; u) = \bar{F}(p; u)$.

Suppose the publisher has computed an *opportunity cost* c for selling this inventory in the exchange; i.e., the publisher stands to gain c if the impression is given to a reservation advertiser. Given opportunity cost $c \geq 0$, the publisher picks the price that maximizes the expected revenue by solving the optimization problem $R(c; u) = \max_{p \geq 0} \bar{F}(p; u)p + F(p; u)c$. Changing variables, we define $r(s; u) = s\bar{F}^{-1}(s; u)$ to be the expected revenue under acceptance probability s and user information u , and rewrite this as²

$$R(c; u) = \max_{s \in [0, 1]} r(s; u) + (1 - s)c. \quad (1)$$

Also, let $s^*(c; u)$ be the least maximizer of (1), and $p^*(c; u) = \bar{F}^{-1}(s^*(c; u); u)$ be the price that verifies the maximum.

ASSUMPTION 1. The expected revenue $r(s; u)$ is continuous in s , concave in s , nonnegative, and bounded and satisfies $\lim_{s \rightarrow 0} r(s; u) = 0$ for every user information $u \in \mathcal{U}$. We call a function that satisfies all of the assumptions above a regular revenue function.

These assumptions are common in RM literature (see, e.g., Gallego and van Ryzin 1994). A sufficient condition for the concavity of the revenue is that bids have increasing failure rates (Lariviere 2006). Regularity implies, among other things, the existence of a *null price* $p_\infty(u)$ such that $\lim_{p \rightarrow p_\infty(u)} \bar{F}(p; u)p = 0$. Additionally, it allows us to characterize the value function $R(c; u)$. In §EC.1.1 of the online appendix, we show that the revenue function remains regular in the presence of multiple bidders in AdX by considering the joint density of the highest and second-highest bids.

PROPOSITION 1. Suppose that r is regular revenue function. Then, for fixed user information u we have that $R(c; u)$ is nondecreasing in c , convex in c , and continuous in c and $R(c; u) \geq c$. Additionally, $R(c; u) - c$ is nonincreasing in c , $s^*(c; u)$ is nonincreasing in c , and $p^*(c; u)$ is nondecreasing in c .

An important consequence of the above is that the maximum revenue expected from submitting an impression to AdX is always greater than the opportunity cost. This should not be surprising, since the publisher can pick a price high enough to compensate for the revenue loss of not assigning the impression. Hence, assigning an impression directly to an advertiser (rather than first testing the exchange) is never the right decision, and in Figure 1 the upper branch is never taken.

² We define the generalized inverse distribution function as $\bar{F}^{-1}(s; u) = \inf\{p \geq 0: \bar{F}(p; u) \leq s\}$ to take into account the case where the distribution is not absolutely continuous.

2.3. Discussion of the Assumptions

In the absence of a fixed cost, the optimal policy tests the exchange before assigning the impression to the contracts. Such a result depends strongly on the publisher's ability to dynamically adjust the reserve to take into account the opportunity cost of "losing" an impression of high quality to the exchange. If lacking the ability to price dynamically, then the publisher would only test the exchange when the expected revenue from AdX exceeds the contracts' opportunity cost. In §6.3 we numerically study the performance of a static-price policy that sets a constant reserve price throughout the horizon.

Publishers usually receive a revenue share of all impressions sold in the exchange. Under such a *revenue-sharing scheme*, the exchange keeps a fraction α of the bidder's payment p , and the publisher receives the amount $(1 - \alpha)p$ for the impression. Our model can accommodate this scheme by increasing the impression's opportunity cost to $c/(1 - \alpha)$. It is straightforward to show that the publisher's AdX value function is now given by $R_\alpha(c; u) = (1 - \alpha)R(c/(1 - \alpha); u)$ and the optimal price is $p_\alpha^*(c; u) = p^*(c/(1 - \alpha); u)$, where R and p^* denote the value function and optimal price in the case of no revenue sharing, respectively.

Publishers typically are not charged a fixed cost each time they access AdX. However, a publisher may still assign the exchange a fixed cost $\ell > 0$ to take into account, for example, the negative effect of latency in the user experience or the opportunity cost of capacity when bandwidth is limited. In this case, the publisher would access the exchange only if the marginal expected contribution from the exchange exceeds the fixed cost, that is, $R(c; u) - c \geq \ell$. In view of Proposition 1, the marginal expected contribution $R(c; u) - c$ is nonincreasing in c , and one can show that the publisher accesses the exchange only if the opportunity cost is less than or equal to the threshold $c^*(\ell; u) = \sup\{c: R(c; u) - c \geq \ell\}$. When the opportunity cost is higher than the threshold, the publisher stands to gain little from accessing the exchange and decides to bypass the spot market in the presence of the fixed cost.

Two final assumptions of our model, which are pervasive in the RM literature, are the stationary quality and independence of the user arrival process. The former assumption is not entirely realistic, because traffic patterns typically vary through the day. For example, an online newspaper may observe a spike of traffic in the mornings from office users and another in the night from home users. Our model can accommodate nonstationary traffic patterns in a straightforward way by allowing the distributions of placement qualities and bids to be time dependent as done in Talluri and van Ryzin (1998). The latter assumption is not very restrictive because unique users visiting a website

arrive essentially at random, so one should expect intertemporal correlation to be weak.

3. Problem Formulation

In this section we start by formulating an optimal control policy for yield maximization based on dynamic programming (DP), and then we proceed to discuss the impact on the publisher's problem of positive correlation between the placement quality of the contracts and the bids from the exchange.

3.1. Dynamic Programming Formulation

Let $(n, x) \in \mathbb{Z} \times \mathbb{Z}^A$ be the state of the system, where we denoted by n the total number of impressions remaining to arrive, and by $x = \{x_a\}_{a \in \mathcal{A}}$ the number of impressions needed to comply with each advertiser's contract. Let the value function, denoted by $J_n(x)$, be defined as the optimal expected yield obtainable under state (n, x) . Using the fact that it is optimal to first test the exchange, we obtain the following Bellman equation:

$$\begin{aligned} J_n(x) = & \mathbb{E}_{U_n} \left[\max_{p \geq 0} \left\{ \bar{F}(p; U_n)(p + J_{n-1}(x)) \right. \right. \\ & \left. \left. + (1 - \bar{F}(p; U_n)) \max_{a \in \mathcal{A}_0} \{ \gamma Q_{n,a} + J_{n-1}(x - \mathbf{1}_a) \} \right\} \right] \\ = & J_{n-1}(x) \\ & + \mathbb{E}_{U_n} \left[R \left(\max_{a \in \mathcal{A}_0} \{ \gamma Q_{n,a} - \Delta_a J_{n-1}(x) \}; U_n \right) \right], \quad (2) \end{aligned}$$

where we define $\mathbf{1}_a$ as a vector with a one in entry a and zero elsewhere, $\mathbf{1}_0 = 0$, and $\Delta_a J_n(x) = J_n(x) - J_n(x - \mathbf{1}_a)$ as the expected marginal yield of one extra impression for advertiser a . In (2) the objective accounts for the yield obtained from attempting to send the impression to AdX. The first term in the outer "maximand" accounts for the expected revenue from the exchange, and the second term accounts for the decision of assigning the impression to a reservation or discarding it (when $a = 0$). In (2) we use the fact that assigning an impression directly to an advertiser is never the right decision (except in boundary conditions; see below). The publisher, however, may choose to discard impressions with low quality after rejection by AdX.

Our objective is to compute $J^* = J_N(C)$. Let M be an upper bound on the expected yield.³ The boundary conditions are $J_n(x) = -M$ for all x such that (s.t.) $x_a < 0$ for some $a \in \mathcal{A}$, and $J_n(x) = -M$ for all $n < \sum_{a \in \mathcal{A}} x_a$. Recall that when the contract with an advertiser is fulfilled, no extra yield is obtained from assigning more impressions to that advertiser. This is the case of the first boundary condition, which guarantees that

³ One could set, for example, $M \triangleq N \max\{p_\infty, \gamma \bar{Q}\}$, where $p_\infty = \max_{u \in \mathcal{U}} p_\infty(u)$ and \bar{Q} is an upper bound on the placement quality.

advertisers whose contract is fulfilled are excluded from the assignment. In particular, when $x = 0$, all remaining impressions are sent to AdX with the yield-maximizing price $p^*(0)$. The second boundary condition guarantees that contracts are always fulfilled. When $\sum_{a \in \mathcal{A}} x_a = n$, AdX must be bypassed, and impressions should be assigned directly to the advertisers. The optimal policy is described in Policy 1.

Policy 1 (Optimal Dynamic Programming Policy)

- 1: Observe state (n, x) , and the vector of attributes u_n .
- 2: Determine the vector of placement qualities q_n .
- 3: Let $a_n = \arg \max_{a \in \mathcal{A}_0} \{\gamma q_{n,a} - \Delta_a J_{n-1}(x)\}$.
- 4: Submit to AdX with price $p^*(\gamma q_{n,a_n} - \Delta_{a_n} J_{n-1}(x); u_n)$.
- 5: if the impression rejected by AdX and $a_n \neq 0$ then
- 6: Assign to advertiser a_n .
- 7: end if

When the impression is submitted to AdX, the optimal price ponders an opportunity cost of $\gamma q_{n,a_n} - \Delta_{a_n} J_{n-1}(x)$. This opportunity cost, when positive, is just the value of the impression adjusted by the loss of potential yield from assigning the impression right now. Note that the two boundary conditions are implicit in the optimal policy. This guarantees that the policy complies with the contracts. It is routine to check that the value function $J_n(x)$ is finite for all feasible states and that Policy 1 is optimal for the dynamic program in (2). It is worth noting that to implement the optimal policy one needs to precompute the value function, which is intractable in most real instances.

3.1.1. AdX Revenue as Primary Objective. To gain some insights into the structure of the problem, it is of interest to consider the limiting choices of $\gamma = 0$ and $\gamma = \infty$, which can be understood as the solution of a bilevel approach to the problem.⁴ In the first case the publisher's primary problem amounts to maximizing the revenue from AdX subject to the constraint that the total number of impressions sold in the exchange is less than $N - \sum_{a \in \mathcal{A}} C_a$, which guarantees that there are enough impressions remaining to satisfy the contracts. The resulting primary problem is a traditional one-product dynamic pricing problem, as in Gallego and van Ryzin (1994). The secondary problem amounts to assigning the impressions rejected by the exchange to the contracts so as to maximize placement quality.

3.1.2. Contract Quality as Primary Objective. The limiting case of $\gamma = \infty$ corresponds to the publisher's prioritizing the quality of the impressions assigned to contracts and submitting the remanent inventory to AdX. In this case, the primary problem amounts to

maximizing the quality of the impressions assigned and is similar to the capacity allocation problem in Talluri and van Ryzin (1998). The secondary problem involves maximizing the revenue from AdX by picking a reserve for those impressions that are not assigned to any contract. The publisher prices all remnant impressions according to the revenue maximizing price, collecting in the process an expected revenue of $(N - \sum_{a \in \mathcal{A}} C_a)R(0)$.

3.2. Impact of User Attributes

Publishers typically disclose some of the user attributes to the exchange, which allows advertisers to bid strategically based on this information. Similar targeting criteria across both channels can result in *positive correlation* between the placement quality of the contracts and the bids from the exchange. To obtain some managerial insights on the impact of user information, in the remainder of this section we discuss the effect of correlation on the publisher's joint allocation problem.

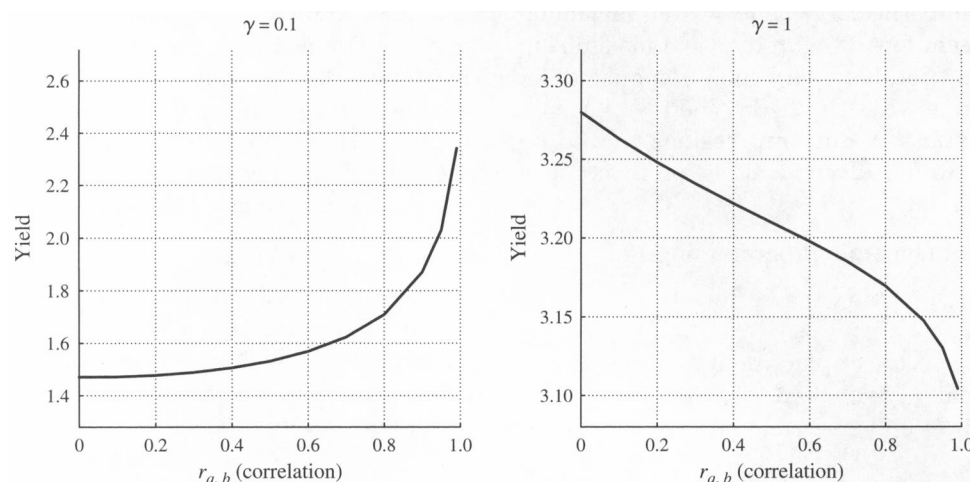
Positive correlation creates two interdependent effects: a *diversification loss effect* and a *price discrimination effect*. The former is a negative effect. The benefit of jointly optimizing over both channels is derived, to a great extent, from the publisher's ability to exploit the exchange to extract rent from impressions that are less attractive to the guaranteed contracts. This diversification effect is severely undermined when the targeting criteria in both channels are in perfect synchrony, and advertisers compete for the same inventory. The latter is a positive effect. With imperfect knowledge about AdX's bids, the publisher cannot fully extract the AdX surplus. However, in the presence of correlation, the publisher can exploit the placement quality (or user attributes) as a covariate to predict bids, adjust the reserve price accordingly, and extract a higher surplus from the exchange.

The total contribution of these antagonistic effects is indeterminate, and in some cases the price discrimination effect may even dominate, resulting in yield increasing with correlation. Figure 3 plots the expected yield as a function of correlation for a publisher with one contract and one bidder in the exchange. Notably, when the publisher prioritizes the contracts (γ is high), yield decreases with correlation. However, when the publisher assigns a higher priority to the revenue from AdX (γ is low), yield increases with correlation. There the detrimental consequences of positive correlation on channel diversification are compensated, to some extent, by the publisher's ability to price more effectively in the exchange by exploiting the user attributes.

As a final remark, there are several reasons why the correlation between these channels might not be perfect. First, publishers usually do not disclose all user attributes to the spot market, thus rendering the targeting in the latter coarser. For example, the publisher might use registered users' personal information

⁴ We thank the associate editor for this suggestion.

Figure 3 (Color online) Publisher's Expected Yield as a Function of the Correlation $r_{q,b}$ for Trade-off Parameters $\gamma = 0.1, 1$



Notes. The publisher in consideration has signed one contract with capacity $\rho = 0.6$ and has one bidder in the exchange. The distribution of the placement quality and the bid from the exchange is a bivariate log-normal distribution with means $\mu_q = \mu_b = 1$, variances $\sigma_q^2 = \sigma_b^2 = \frac{1}{2}$, and correlation $r_{q,b}$. The solid curve denotes the publisher's yield.

to improve the targeting of guaranteed contracts, but not disclose this information to AdX because of privacy concerns. Second, advertisers in the spot market are increasingly targeting users based on *cookies*, which are private bits of information stored in the users' computers that allow user behavior on the Web to be tracked. Cookies are dropped by third-party advertisers when users visit websites and are accessible only to members of that advertising network. Thus, a strong component of spot market bids is based on private information. In §5.3 we empirically explore the correlation between the bids from the exchange and the placement quality and show that the dependence is statistically weak.

4. Deterministic Approximation

Unfortunately, the state space of the DP in (2) has size $O(N^{A+1})$, and in most real-world problems the number of impressions in a single horizon can be in the order of millions. So the DP is not efficiently solvable. We give, instead, an approximation in which stochastic quantities are replaced by their expected values, and are assumed to be continuous. Such deterministic approximation problems (DAP) are popular in RM (see, e.g., Talluri and van Ryzin 1998). In our setting, the approximation we make is to enforce contracts to be satisfied only in expectation. We formulate the problem based on this assumption and obtain an infinite-dimensional program. This DAP is solved by considering its dual problem, which turns out to be a more tractable finite-dimensional convex program. Finally, we wrap a full stochastic policy around it (one that always meets the contracts, not just in expectation) and show that this policy is asymptotically optimal when the number of impressions and capacity are scaled up proportionally.

4.1. Formulation of Deterministic Problem

We aim for an approximation in which (i) the policy is independent of the history but dependent on the realization of the vector of attributes u_n (recall that placement qualities are deterministically determined based on the attributes), (ii) capacity constraints are met in expectation, and (iii) controls are allowed to randomize. These approximations turn out to be reasonable when the number of impressions is large. When an impression arrives, the publisher controls the reserve price submitted to AdX and the advertiser to whom the impression is assigned, if rejected by AdX. Alternatively, in this formulation we state the controls in terms of total probabilities, where each control is a function from the attribute space \mathcal{U} to $[0, 1]$. Let $\vec{s} = \{s_n(\cdot)\}_{n=1,\dots,N}$ and $\vec{i} = \{i_n(\cdot)\}_{n=1,\dots,N}$ be vectors of functions from \mathcal{U} to \mathbb{R} , such that when the n th impression arrives with attributes u the impression is accepted by AdX with probability $s_n(u)$, and with probability $i_{n,a}(u)$ it is assigned to advertiser a . From these controls, one can determine the conditional probability of an impression's being assigned to advertiser a given that it has been rejected by AdX by $I_{n,a}(u) = i_{n,a}(u)/(1 - s_n(u))$, and the reserve price to be posted in the exchange by $\bar{F}^{-1}(s_n(u); u)$. When it is clear from the context, we simplify notation by eliminating the dependence on u from the controls.

A control is feasible for the DAP if (i) it satisfies the contractual constraints in expectation, (ii) the individual controls are nonnegative, and (iii) for every realization of the qualities the probabilities sum up to at most one. We denote by \mathcal{P} the set of controls for one impression that satisfy the latter two conditions. That is, $\mathcal{P} = \{(s, i) \in \mathcal{U} \rightarrow [0, 1]^{A+1} : \sum_{a \in \mathcal{A}} i_a + s \leq 1, s \geq 0, i \geq 0\}$. The objective of the DAP is to find a sequence of real-valued

measurable functions that maximize the expected yield or, equivalently,

$$J^D \triangleq \max_{(s_n, i_n) \in \mathcal{P}} \mathbb{E} \left[\sum_{n=1}^N r(s_n; U_n) + \gamma \sum_{a \in \mathcal{A}} i_{n,a} Q_{n,a} \right] \quad (3)$$

$$\text{s.t. } \mathbb{E} \left[\sum_{n=1}^N i_{n,a} \right] = C_a, \quad \forall a \in \mathcal{A}.$$

The first term of the objective accounts for the revenue from AdX, while the second accounts for the quality perceived by the advertisers. In order to compute the DAP's optimal solution, we consider its dual problem, which we derive next.

4.1.1. Derivation of the Dual Problem to DAP.

To find the dual function, we introduce Lagrange multipliers $v = \{v_a\}_{a \in \mathcal{A}}$ for the capacity constraints (3). The Lagrangian, denoted by $\mathcal{L}(\tilde{s}, \tilde{i}; v)$, is

$$\mathcal{L}(\tilde{s}, \tilde{i}; v) = \mathbb{E} \left[\sum_{n=1}^N r(s_n; U_n) + \gamma \sum_{a \in \mathcal{A}} i_{n,a} Q_{n,a} - \sum_{a \in \mathcal{A}} v_a (i_{n,a} - \rho_a) \right],$$

where we used the fact that $C_a = N\rho_a$. The dual function, denoted by $\psi(v)$, is the supremum of the Lagrangian over the set \mathcal{P} . Thus, we have that

$$\begin{aligned} \psi(v) &= \sup_{(s_n, i_n) \in \mathcal{P}} \mathcal{L}(\tilde{s}, \tilde{i}; v) \\ &= N \sup_{(s, i) \in \mathcal{P}} \mathbb{E} \left[r(s; U) + \gamma \sum_{a \in \mathcal{A}} i_a Q_a - \sum_{a \in \mathcal{A}} v_a (i_a - \rho_a) \right] \\ &= N \sup_{s \geq 0} \left\{ \mathbb{E}[r(s; U)] \right. \\ &\quad \left. + \sup_{i \geq 0, \sum_{a \in \mathcal{A}} i_a \leq 1-s} \mathbb{E} \left[\sum_{a \in \mathcal{A}} i_a (\gamma Q_a - v_a) \right] \right\} + N \sum_{a \in \mathcal{A}} v_a \rho_a \\ &= N \sup_{s \geq 0} \mathbb{E} \left[r(s; U) + (1-s) \max_{a \in \mathcal{A}_0} \{\gamma Q_a - v_a\} \right] \\ &\quad + N \sum_{a \in \mathcal{A}} v_a \rho_a \\ &= N \mathbb{E} \left[R \left(\max_{a \in \mathcal{A}_0} \{\gamma Q_a - v_a\}; U \right) \right] + N \sum_{a \in \mathcal{A}} v_a \rho_a, \end{aligned}$$

where the second equation follows from the fact that the objective is separable in the impressions and the stationarity of the user attributes, the third from partitioning the optimization between AdX acceptance and the assignment probability controls, the fourth from optimizing over the advertiser assignment controls i , and the last equation from solving the AdX variational problem. Note that R is convex and nondecreasing in its first argument and the maximum is convex with

regard to (w.r.t) v , hence the composite function within the expectation is convex. Using the fact that expectation preserves convexity, we obtain that the objective $\psi(v)$ is convex in v .

Next, the dual problem is $\inf_v \psi(v)$. When the revenue function is regular, the DAP's objective is concave and bounded from above. Moreover, the constraints of the primal problem are linear, and the feasible set \mathcal{P} is convex. Hence, by the Strong Duality Theorem (Luenberger 1969, p. 224), the dual problem attains the primal objective value, and the dual problem is given by the following convex stochastic problem:

$$J^D = N \min_v \left\{ \mathbb{E} \left[R \left(\max_{a \in \mathcal{A}_0} \{\gamma Q_a - v_a\}; U \right) \right] + \sum_{a \in \mathcal{A}} v_a \rho_a \right\}. \quad (4)$$

4.1.2. Deterministic Optimal Control. Once the optimal dual variables v are known, the primal solution can be constructed from plugging the optimal Lagrange multipliers into $\mathcal{L}(\tilde{s}, \tilde{i}; v)$. Following the derivation of the dual problem, we obtain that the optimal survival probability is $s^*(\max_{a \in \mathcal{A}_0} \{\gamma q_a - v_a\}; u)$. Hence, the impression has a value of $\max_{a \in \mathcal{A}_0} \{\gamma q_a - v_a\}$ for the publisher, who picks the reserve price that maximizes revenue given that value. From the optimization over the assignment controls, we see that an impression is assigned to an advertiser a only if the *contract-adjusted quality* $\gamma q_a - v_a$ is maximized, where the dual variables v_a act as the *bid prices* of the guaranteed contracts. Additionally, the impression can be discarded only if the maximum is not verified by an advertiser (i.e., all contract-adjusted qualities are nonpositive). Finally, the resulting control is stationary; that is, the control is independent of the number of impressions left in the horizon.

Note that optimizing the Lagrangian does not specify how the impression should be assigned when multiple advertisers attain the maximum. In the case when the probability of a tie occurring is zero, the problem admits a simple solution: assign the impression to the unique maximizer of $\gamma q_a - v_a$. We formalize this discussion in the following theorem.

THEOREM 1. Suppose that the revenue function is regular and that there is zero probability of a tie occurring; i.e., $\mathbb{P}\{\gamma Q_a - v_a = \gamma Q_{a'} - v_{a'}\} = 0$ for all distinct $a, a' \in \mathcal{A}_0$. Then, the optimal controls for the DAP are $s(u) = s^*(\max_{a \in \mathcal{A}_0} \{\gamma q_a - v_a\}; u)$, and $I_a(u) = \mathbf{1}\{\gamma q_a - v_a > \gamma q_{a'} - v_{a'} \forall a' \in \mathcal{A}_0\}$; that is, the impression is assigned to the unique advertiser maximizing the contract-adjusted quality. Furthermore, the optimal dual variables solve the equations

$$\mathbb{E}[(1 - s^*(\gamma Q_a - v_a; U)) \cdot \mathbf{1}\{\gamma Q_a - v_a > \gamma Q_{a'} - v_{a'} \forall a' \in \mathcal{A}_0\}] = \rho_a, \quad \forall a \in \mathcal{A}.$$

4.2. The Stochastic Policy

The solution of the DAP suggests a policy for the stochastic control problem, but we must deal with two technical issues: (i) when more than one advertiser maximizes $\gamma q_a - v_a$, we need to decide how to break the tie; and (ii) we are only guaranteed to meet the contracts in expectation, whereas we must meet them exactly. We defer the first issue until §4.3, where we give an algorithm for generalizing the controls to the case where ties are possible.

We propose a bid-price control extended with a pricing function for AdX given by p^* . The policy, which we denote by μ^B , is defined in Policy 2, wherein we let v be the optimal solution of (4). Impressions are only assigned to advertisers with contracts that have yet to be fulfilled. When all contracts are fulfilled, impressions are sent to AdX with the revenue-maximizing price $p^*(0; u_n)$. Moreover, when the total number of impressions necessary to fulfill the contracts, the exchange is bypassed and all incoming impressions are directly assigned to advertisers (no impression is discarded). Hence, the stochastic policy μ^B satisfies the contracts for every sample path.

Policy 2 (Bid-Price Policy with Dynamic Pricing μ^B)

- 1: Observe state (n, x) , and the vector of attributes u_n .
- 2: Determine the vector of placement qualities q_n .
- 3: Let $\mathcal{A}_n = \{a \in \mathcal{A} : x_{n,a} > 0\}$ be the set of ads yet to be satisfied.
- 4: **if** $\sum_{a \in \mathcal{A}} x_{n,a} < n$ **then**
- 5: Let $a_n^* = \arg \max_{a \in \mathcal{A}_n \cup \{0\}} \{\gamma q_{n,a} - v_a\}$.
- 6: Submit to AdX with price $p^*(\gamma q_{n,a_n^*} - v_{a_n^*}; u_n)$.
- 7: **if** impression rejected by AdX and $a_n^* \neq 0$ **then** assign to advertiser a_n^* **else** discard.
- 8: **else**
- 9: Assign to advertiser $a_n^* = \arg \max_{a \in \mathcal{A}_n} \{\gamma q_{n,a} - v_a\}$.
- 10: **end if**

The proposed stochastic policy shares some resemblance to the optimal dynamic programming policy. The intuition is that, when the number of impressions is large, the actual state of the system becomes irrelevant because $\Delta_a J_{m-1}(x)$ is approximately constant (for states in likely trajectories) and equal to v_a . In that case both policies are equivalent.

The policy can be alternatively interpreted as the publisher bidding on behalf of the guaranteed contracts in a sequence of repeated auctions run by the exchange as in Ghosh et al. (2009). The pricing function and the bid prices determine a reserve price or “bid” for the contracts that takes into account the value of assigning the impression to a reservation together with the option value of future opportunities. In this dual interpretation the spot market lies in the spotlight and the guaranteed

contracts are pushed to the background, in sharp contrast to the current practice of first aiming to fulfill the reservations and then submitting the remnant inventory to AdX. Our original interpretation is more appealing because it does not rely so heavily on the publisher always testing the exchange, which may not be optimal, for example, in the presence of a fixed cost.

4.3. Handling Ties

Theorem 1 assumes that there are no ties between advertisers verifying the maximum contract-adjusted quality. In this section we show how to construct a primal optimal solution to the DAP and the corresponding stochastic policy in the general case (e.g., when the distribution of placement quality is discrete or has atoms). Devanur and Hayes (2009) proposed introducing small random and independent perturbations to the qualities, or smoothing the dual problem to break ties. We provide an alternate method that directly attacks ties and provides a randomized tie-breaking rule. Computing the parameters of the tie-breaking rule requires solving an assignment problem on a graph of size at most 2^A . In §6 we argue that, in practice, the number of ties is roughly linear in the number of advertisers and the problem is tractable. Additionally, by failing to take ties into account, the publisher can incur significant losses.

A modified bid-price solution with tie-breaking for the DAP is defined by a pair (v, p) with $v \in \mathbb{R}^A$ the vector of dual variables and $p: 2^{\mathcal{A}_0} \rightarrow [0, 1]^{A+1}$ the tie-breaking probabilities. For any nonempty subset $S \subseteq \mathcal{A}_0$, the tie-breaking probability $p_a(S)$ determines the probability that the impression is assigned to contract $a \in \mathcal{A}_0$ when the maximum is verified exactly by all the advertisers in S , and the impression is rejected by AdX. In the case that exactly one advertiser verifies the maximum the tie is a singleton. Given a vector of dual variables $v \in \mathbb{R}^A$, the tie-breaking probabilities are obtained by solving the following assignment problem:

$$\sum_{S \subseteq \mathcal{A}_0: a \in S} \mathbb{P}(S\text{-tie}) p_a(S) = \rho_a, \quad \forall a \in \mathcal{A}, \quad (5a)$$

$$\sum_{a \in S} p_a(S) = 1, \quad \forall S \subseteq \mathcal{A}_0, \quad (5b)$$

$$p_a(S) \geq 0, \quad \forall S \subseteq \mathcal{A}_0, a \in S, \quad (5c)$$

where we let $\mathbb{P}(S\text{-tie}) = \mathbb{E}[(1 - s^*(\max_{a \in \mathcal{A}_0} \{\gamma Q_a - v_a\}; U)) \mathbf{1}\{S = \arg \max_{a \in \mathcal{A}_0} \{\gamma Q_a - v_a\}\}]$ be the probability that the maximum is verified exactly by all the advertisers in S , and the impression is rejected by AdX. Equation (5a) guarantees that contract a is assigned exactly ρ_a impressions, and Equation (5b) guarantees that for each tie S the probabilities sum up to one.

An important question is whether the previous problem admits a feasible solution. The next result

proves that the answer is affirmative when the dual variables v are optimal for the dual problem (4), and characterizes the optimal solution for the DAP.

PROPOSITION 2. Suppose that $v \in \mathbb{R}^A$ is an optimal solution for the dual problem (4). Then, there exists a tie-breaking rule $p: 2^{\mathcal{A}_0} \rightarrow [0, 1]^{A+1}$, solving problem (5). Additionally, the optimal solution for the DAP prices in AdX according to the survival probability $s(u) = s^*(\max_{a \in \mathcal{A}_0} \{\gamma q_a - v_a\}; u)$ and, if rejected, assigns the impression to contract a with probability $I_a(u) = \sum_{S \subseteq \mathcal{A}_0: a \in S} p_a(S) \mathbf{1}\{S = \arg \max_{a' \in \mathcal{A}_0} \{\gamma q_{a'} - v_{a'}\}\}$.

The proof proceeds by casting the problem as a maximum flow problem in a bipartite graph and then exploits the optimality conditions of v in the dual problem to lower bound every cut in the bipartite graph. The feasibility of the proposed solution follows from constraints (5a) and (5b). To prove optimality it suffices to show that it attains the dual objective value or that the proposed solution satisfies the complementary slackness conditions. The latter follows trivially.

Once the optimal controls are calculated, we construct our stochastic policy as follows. When $\sum_{a \in \mathcal{A}} x_{n,a} < n$ we let $S_n = \arg \max_{a \in \mathcal{A}_n \cup \{0\}} \{q_{n,a} - v_a\}$ be the set of advertisers that attain the maximum within advertisers with contracts that have yet to be fulfilled. Now, if the impression is rejected by AdX, we assign it to advertiser $a \in S_n$ with probability $p_a(S_n)$. Similar corrections as before are applied when the total number of impressions left is equal to the number of impressions necessary to fulfill the contracts.

4.4. Asymptotic Analysis

In this section we show that the heuristic policy constructed from the DAP is asymptotically optimal for the stochastic problem when the number of impressions and capacities are scaled up proportionally. To this end we extend the analysis of Gallego and van Ryzin (1994) and Talluri and van Ryzin (1998) to capture the multiobjective nature of our problem. We prove that any Pareto-optimal policy of the stochastic control can be approximated by a simple modified bid-price policy that performs asymptotically close both in terms of the AdX revenue and contract quality. We prove this result in two steps.

The first result (Proposition 3) shows that every policy of the stochastic control problem is Pareto dominated by a solution to the DAP, which implies that the efficient frontier of the stochastic control problem is dominated by that of the DAP. Because the DAP is convex, its efficient frontier is concave, and the publisher can achieve any Pareto-efficient point by picking a suitable trade-off parameter γ . Let J_A^μ and J_C^μ be the expected AdX revenue and contract quality associated with a stochastic policy μ . Let $J_A^{D(\gamma)}$ and $J_C^{D(\gamma)}$ be the expected AdX revenue and contract quality of the DAP for trade-off parameter $\gamma \geq 0$. We have the following result.

PROPOSITION 3. For every feasible stochastic policy μ there exists a trade-off parameter $\gamma \geq 0$ such that $J_A^\mu \leq J_A^{D(\gamma)}$ and $J_C^\mu \leq J_C^{D(\gamma)}$.

The proof of the previous result is as follows. First, we proceed by taking any feasible stochastic control policy and constructing a feasible solution for the DAP by taking expectations over the history. Later, we exploit the concavity of the objective and apply Jensen's inequality to show that this new solution attains a greater revenue and quality in the DAP. We conclude by showing that the efficient frontier of the DAP is concave, and then invoking a supporting hyperplane argument to prove that any Pareto point can be achieved by optimizing a weighted combination of the objectives.

The second result (Theorem 2) shows that for any trade-off parameter γ , the performance of the modified bid-price policy derived from the respective optimal solution of the DAP is asymptotically close, both in terms of AdX revenue and contract quality, to that predicted by the DAP. In other words, these results imply that the efficient frontier achieved by the proposed heuristic is asymptotically "close" to that of the stochastic control problem. In the following, let $J^{B(\gamma)} = J_A^{B(\gamma)} + \gamma J_C^{B(\gamma)}$ be the expected yield, $J_A^{B(\gamma)}$ the expected AdX revenue, and $J_C^{B(\gamma)}$ the expected contract quality under the stochastic policy $\mu^{B(\gamma)}$.

THEOREM 2. Fix the trade-off parameter γ . Let $K = \sqrt{(A/(A+1)) \sum_{a \in \mathcal{A}_0} ((1 - \rho_a)/\rho_a)}$. Then, the expected performance of $\mu^{B(\gamma)}$ relative to the objective value of the deterministic approximation is lower bounded

- (i) in terms of AdX revenue by $J_A^{B(\gamma)}/J_A^{D(\gamma)} \geq 1 - K/\sqrt{N}$,
- (ii) in terms of contract quality by $J_C^{B(\gamma)}/J_C^{D(\gamma)} \geq 1 - K/\sqrt{N}$, and
- (iii) in terms of yield by $J^{B(\gamma)}/J^{D(\gamma)} \geq 1 - K/\sqrt{N}$.

Fix the capacity-to-impression ratio of each advertiser, and consider a sequence of problems in which capacity and impressions are scaled up proportionally according to ρ . Then the two previous results imply that the performance under policy $\mu^{B(\gamma)}$ converges to the performance of the optimal online policy as N goes to infinity. In proving the previous bounds, we look at N^* , the first time that any advertiser's contract is fulfilled or the point is reached where all arriving impressions need to be assigned to the advertisers. We refer to the time after N^* as the *leftover regime*. The first key observation in the proof is that before time N^* , the controls of the stochastic policy behave exactly as the optimal deterministic controls, and the expected performance of the policy coincides with that predicted by the DAP. The second key observation is that the expected number of impressions in the leftover regime is $O(\sqrt{N})$, and the leftover regime has a small impact on the objective. In fact, using a

Chernoff bound, it is straightforward to show that the probability that the number of impressions in the leftover regime exceeds a fixed fraction of the total impressions decays exponentially fast.

The policy described in §4.2 is stationary in the sense that it does not react to changes in supply: the dual variables v and the tie-breaking probabilities p are computed at the beginning and remain fixed throughout the horizon. (Prices in the exchange, however, are dynamically adjusted to account for the opportunity cost of not assigning an impression to a guaranteed contract.) To address this issue, in practice, one would periodically resolve the deterministic approximation (4). Recently, Jasin and Kumar (2012) showed that carefully chosen periodic resolving schemes together with probabilistic allocation controls can achieve bounded yield loss w.r.t. the optimal online policy. It is worth noting that those results do not directly apply to our setting: they consider a network RM problem with discrete choice, whereas our model deals with jointly distributed (and possibly continuous) placement qualities and AdX. Nevertheless, by periodically resolving the DAP, one should be able to obtain similar performance guarantees for the yield loss of the control.

5. Data Model and Estimation

In order to study the performance of our algorithm on display ads, we introduce a data model based on our observation of actual publisher inventory and then present data sets from seven anonymous publishers. We selected representative publishers of different size: two small publishers with approximately 10 contracts, two large publishers with approximately 100 contracts, and three medium-sized publishers in between. The publishers are mostly online gaming websites and news websites. The data sets were collected over a period of one week during March of 2010, and the number of impressions in each data set ranges from 300 thousand to seven million. Additionally, the fraction of impressions reserved for contracts as given by $\sum_{a \in \mathcal{A}} \rho_a$ varies across publishers, with two publishers highly constrained, one moderately constrained, and the remainder lowly constrained. Publishers’ characteristics are given in Table 2.

5.1. Guaranteed Contracts

We have thus far assumed that any user could be potentially assigned to any advertiser. In practice, however, advertisers have specific targeting criteria. The targeting criteria of the guaranteed contracts is based on the URL, the geographic location, and the type of browser or operating system of the users, and the time of the day, and contextual features of webpages. Instead of grouping user types according to their attributes, we aggregate user types that match the criteria of the same subset of advertisers. This has the advantage of reducing the space of types to a function of the number of advertisers (which is typically small in practice) rather than the number of possible types (which is potentially large). Hence, a user type is characterized by the subset of advertisers $T \subseteq \mathcal{A}$ that are interested in it. In the following, we let \mathcal{T} be the support of the type distribution, and $\pi(T)$ the probability of an arriving impression being of type T .

The measure of placement quality for a contract is the predicted click-through rate to each impression, which is learned via a system that uses the impression’s attributes and features of the ad creative (such as the text of the ad) as explanatory variables. We refer the reader to McMahan et al. (2013) for a detailed description of the methods employed to predict click-through rates in online advertising. Given a particular type T , the predicted quality perceived by the advertisers within the type is modeled by the nonnegative random vector $Q(T) = \{Q_a(T)\}_{a \in T}$. Thus, the ex ante distribution of quality is given by the mixture of the types’ distribution with mixing probabilities $\pi(T)$. All our previous results hold for the mixture distribution.

Even if the total number of impressions suffices to satisfy the contracts, i.e., $\sum_{a \in \mathcal{A}} \rho_a \leq 1$, the inventory may not be enough to satisfy the contracts’ targeting criteria. Our algorithm guarantees that the total number of impressions C_a is always respected, yet some advertisers may be assigned impressions outside their criteria. If an impression of type T happens to be assigned to an advertiser $a \notin T$, the publisher pays a nonnegative goodwill penalty τ_a . These penalties

Table 2 Characteristics of the Different Publishers

Publisher	Contracts (A)	Types (T)	Capacity ratio ($\sum_a \rho_a$) (%)	Mean quality	Impressions (N)	Acceptance prob. ($s^*(0)$) (%)	Revenue from AdX ($R(0)$)
1	6	10	21	582	1,500,000	68	622
2	12	7	89	47	2,100,000	100	448
3	17	13	43	820	320,000	74	1,883
4	17	15	28	686	930,000	76	1,320
5	29	27	73	1,152	1,800,000	99	1,424
6	98	173	28	542	6,700,000	75	2,076
7	101	406	16	209	7,000,000	67	1,378

Note. The mean quality is defined as the expected quality given that the contract matches were averaged over all contracts; that is, $\sum_a \rho_a \mathbb{E}[Q_a(T)|T \ni a]$.

allow the publisher to prioritize certain reservations, especially when contracts are not feasible.⁵

5.2. Estimation of Placement Qualities

Our policy can be applied both in a parametric and nonparametric fashion. In the former, a parametric model of placement qualities is postulated, the underlying parameters are estimated by using sample data, and the policy's parameters are determined by solving problems (4) and (5) with the estimated distribution. In the latter, problems (4) and (5) are solved directly over sample data by replacing expectations with sample averages. In the remainder of the paper we describe the parametric case, and the nonparametric approach is described in SEC.5 of the online appendix.

Given a particular type T , we observe that the predicted quality perceived by the advertisers within a type is approximately log normal. This can be seen in SEC.3 of the online appendix, where the empirical distribution of log quality is graphically represented for all types of publisher 3. We postulate that quality follows a multivariate log normal with mean vector μ_T and covariance matrix Σ_T for the advertisers in the type and takes a value of $-\tau_a$ for advertisers not in the type. The total distribution of quality is given by the mixture of these types' distribution with mixing probabilities $\pi(T)$. Thus, we have that

$$Q \sim \begin{cases} \ln \mathcal{N}(\mu_T, \Sigma_T) & \text{for } a \in T, \\ -\tau_a & \text{for } a \notin T, \end{cases} \quad \text{w.p. } \pi(T).$$

Logs were analyzed to estimate the types' frequencies, and the parameters of the underlying log-normal distributions (using maximum likelihood estimation). The parameters for the publishers are available as supplemental material at <http://dx.doi.org/10.1287/mnsc.2014.2017> (for confidentiality reasons, all placement qualities were linearly perturbed by a random factor). Table 2 shows the mean placement quality for the publishers in the data set, which varies significantly across publishers.

5.3. Estimation of AdX Bids

Bidding data from the same period of time were used to estimate the primitives of AdX. With multiple bidders, AdX runs a sealed-bid second-price auction. We analyze the first- and second-highest bids for the inventory submitted to AdX *independently* of the impression's attributes and placement qualities. Sample data are used to compute the two primitives of our model: (i) the complement of the quantile of the highest bid $p(s)$, and (ii) the revenue function $r(s)$. Both functions are estimated on a uniform grid $\{s_j\}_1^{100}$ of survival probabilities

in the $[0, 1]$ range as follows. Let $\{(b_{1,m}, b_{2,m})\}_{m=1,\dots,M}$ be the sampled highest and second-highest bids from the exchange. First, for each point in the grid j , the price $p_j = p(s_j)$ is estimated as the $(1 - s_j)$ th population quantile of the highest bid. Then, using sampled bids, we estimate the revenue function w.r.t. to prices at the grid points as

$$r(p_j) = \frac{1}{M} \sum_{m=1}^M \mathbf{1}\{b_{1,m} \geq p_j\} \max\{b_{2,m}, p_j\}. \quad (6)$$

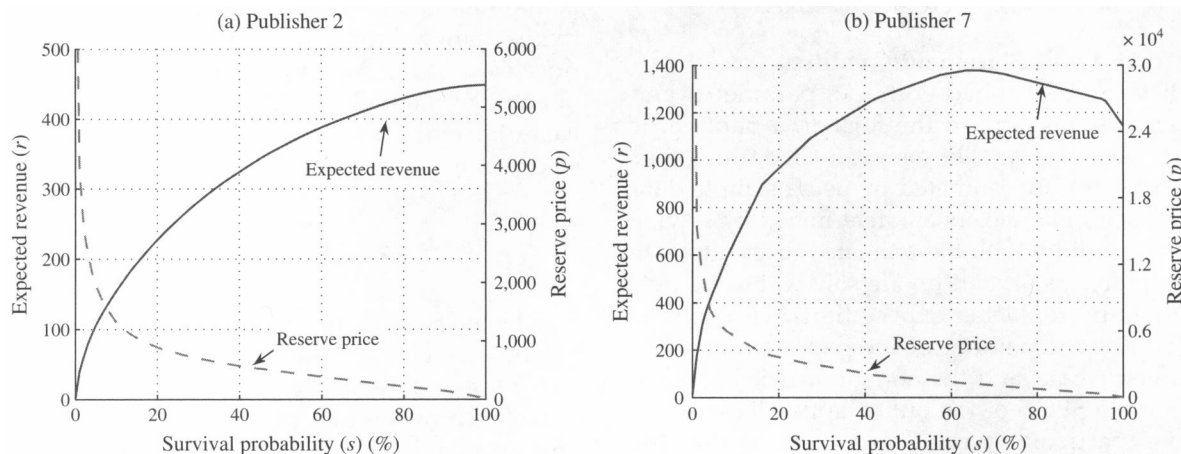
Finally, the revenue function is obtained by composing (6) and $p(s)$. Bids were linearly perturbed by a random factor for confidentiality.

Table 2 provides the optimal revenue $R(0)$ and acceptance probability $s^*(0)$ when the opportunity cost is zero for the different publishers. The distribution of the maximum bid and the resulting optimal revenue function varies significantly across publishers. Interestingly, for some publishers the optimal acceptance probability in the absence of an opportunity cost is close to one, and the publisher is better off setting no reserve price at all. For these publishers, the exchange market is competitive with highly correlated bids, and the probability of not selling the item by setting a reserve price outweighs the benefit of raising the floor price. Figure 4 exhibits the estimated AdX survival probability and revenue function for publisher 2, which has a very high optimal acceptance probability, and publisher 7, which has an optimal acceptance probability in the high 60s.

We provide some insight into the dependence structure between the guaranteed contracts' placement qualities and AdX bids by studying Pearson's correlation between these two quantities over two publishers. The setup is as follows. First, we aggregate impressions by *ad slot*, where ad slot refers to a given position on a publisher's webpage and is defined by the triple $l = (\text{position}, \text{webpage}, \text{publisher})$. The total number of ad slots L is in the order of thousands. Second, we compute the average value of maximum bids and average value of maximum placement qualities (predicted click-through rates) over all impressions corresponding to each ad slot. Letting M_l be the number of impressions in ad slot l , the average value of maximum bids for the slot is $b_l = (1/M_l) \sum_{m \in l} b_{1,m}$, and the average value of maximum placement quality is $q_l = (1/M_l) \sum_{m \in l} \max_{a \in \mathcal{M}} \{q_{m,a}\}$. Finally, we compute the sample correlation coefficient, denoted by $r_{q,b}$, between the vectors of placement qualities $\{q_l\}_{l=1}^L$ and slot bids $\{b_l\}_{l=1}^L$. We find that the correlation of these two vectors is $r_{q,b} \approx -2\%$, and therefore conclude that correlation between the highest bid of an ad slot and the average placement quality is weak. As discussed earlier, this lack of correlation may be the result of advertisers' determining their bids in AdX using different signals

⁵ In SEC.6 of the online appendix, we show that by picking suitably large penalties the publisher can avoid delivering impressions outside the targeting criteria when contracts are feasible.

Figure 4 (Color online) Estimated Survival Probability and Revenue Function for AdX from Two Different Publishers



from the publisher. The usual “lack of correlation does not imply independence” warning must apply here, and this finding should not be interpreted as a statement of independence between these two channels.

6. Experimental Results

In this section we present two numerical experiments conducted to study our model. First, we analyze the impact on the publisher’s yield of jointly optimizing over both channels using actual publisher data. Second, we compare the performance of our policy with those of two popular heuristics. Before discussing the experimental results we present an efficient method to compute the dual variables in the presence of large-scale instances.

6.1. Solution Method

A difficulty of solving the stochastic optimization problem (4) is that the involved multidimensional integral cannot be computed with high accuracy when the publisher has many contracts. We tackle this problem by performing a sample average approximation (SAA), which relies on approximating the underlying stochastic program via sampling, and then solving the approximate problem via a subgradient descent method (SDM).

The basic idea of the SAA is simple: a random sample of placement qualities is generated and the expectation is approximated by the sample average function (Shapiro et al. 2009). Let $\{q_m\}_{m=1}^M$ be an independent and identically distributed (i.i.d.) sample of M vectors of placement qualities, then the SAA is given by

$$\min_v \frac{1}{M} \sum_{m=1}^M R\left(\max_{a \in \mathcal{A}_0} \{\gamma q_{m,a} - v_a\}\right) + \sum_{a \in \mathcal{A}} \rho_a v_a, \quad (7)$$

which is a nondifferentiable convex minimization problem. One can show that the optimal solution and objective value of the SAA problem are consistent estimators of the optimal solution and objective value of

the stochastic program, respectively (see, e.g., Shapiro et al. 2009).

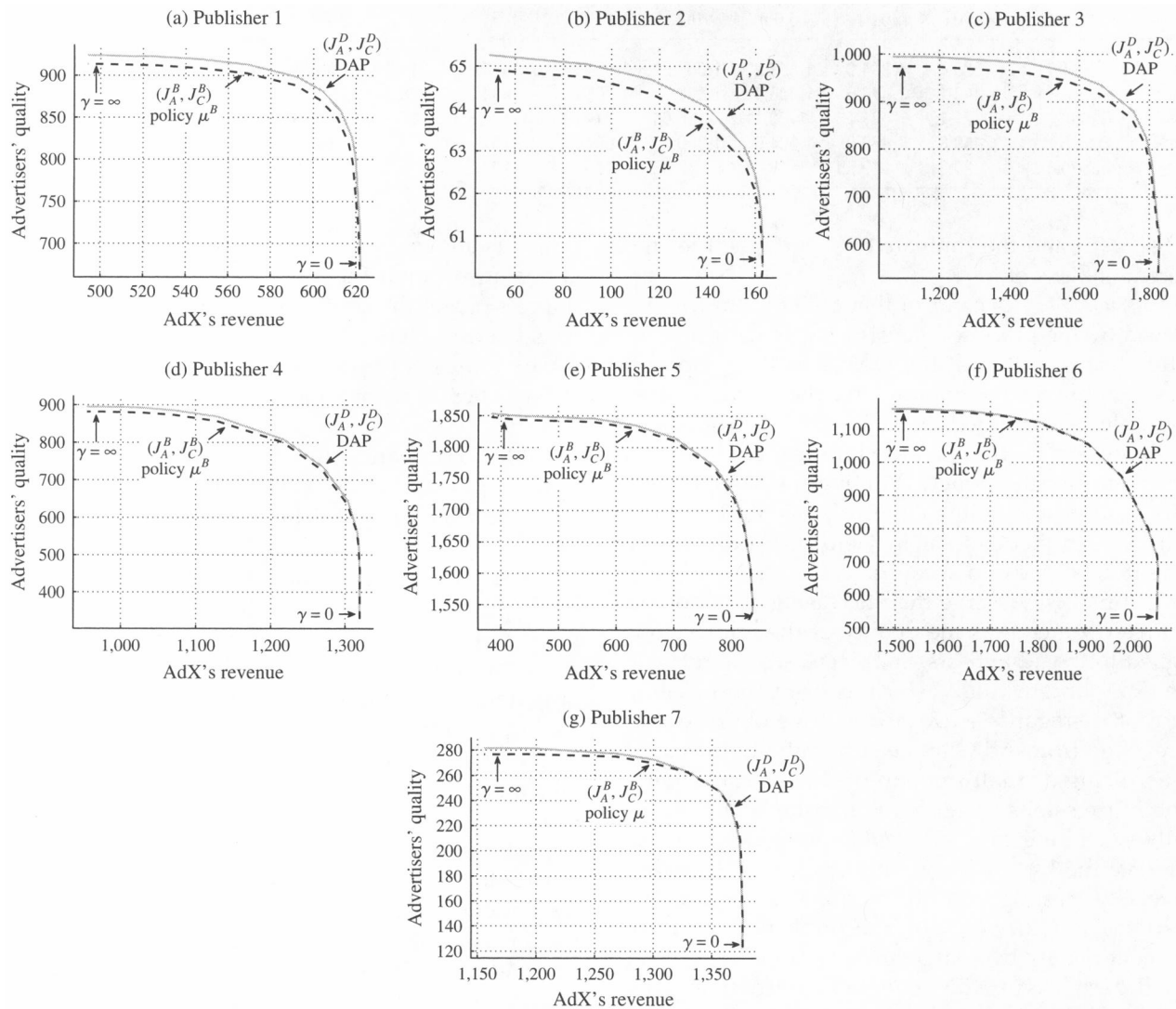
We solve the deterministic SAA problem via a SDM, which involves iterating the dual variables by taking steps on the opposite direction of any subgradient of the approximated objective with a proper step size (see, e.g., Boyd and Mutapcic 2008 for a review on the topic). Starting from an initial solution $v^{(0)}$, our algorithm computes the new dual variables using the formula $v^{(k+1)} = v^{(k)} - \alpha_k g(v^{(k)})$, where $g(v) \in \mathbb{R}^A$ is a subgradient of the SAA objective at point v , and α_k is the step size. (We employ a constant step-length rule; i.e., $\alpha_k = \alpha / \|g(v^{(k)})\|_2$.) A subgradient of the SAA objective function is readily given by $g(v) \triangleq -(1/M) \sum_{m=1}^M (1 - s^*(\gamma q_{m,a_m^*} - v_{a_m^*})) \mathbf{1}_{a_m^*} + \rho$, where $a_m^* \in \arg \max_{a \in \mathcal{A}_0} \{\gamma q_{m,a} - v_a\}$ is any advertiser achieving the maximum in the m th sample.

For a given instance of the problem, the SAA is solved on a *training set* of $M = 10,000$ samples with 2,000 iterations of the SDM, which in total take an average of two minutes on a personal computer⁶. Since both the SAA objective value and gradient can be computed in $O(MA)$ time, the SDM is able to quickly obtain a dual solution. Once a dual solution is obtained, we construct the stochastic control policy by solving the flow problem described in §4.3. In §EC.7 of the online appendix, we argue that one needs to consider at most $O(T + A)$ ties and thus the flow problem has at most $O(TA)$ variables. A certificate of suboptimality of the dual solution is established by constructing a feasible solution to the primal problem of the SAA, and then invoking weak duality to obtain a lower bound on the optimal value of the dual problem.

One advantage of the SAA is that it provides a nonparametric approach to estimate the dual variables when the distribution of the placement quality

⁶ The algorithm is implemented in MATLAB 7.11 and executed on a Windows PC with an Intel 2.0 GHz CPU and 4 GB of RAM.

Figure 5 (Color online) Plots, in a Quality vs. Revenue Graph, of the Efficient Frontier for Seven Publishers



Note. The solid curve denotes the performance of the DAP (J_A^D, J_C^D) , and the dashed curve denotes the performance of simulated policy (J_A^B, J_C^B) for the different choices of γ .

is unknown. In order to solve the original stochastic minimization problem in practice, first one needs to postulate a parametric model for placement quality (as done in §5) and then to use a sample of data to learn the parameters of the underlying model. The SAA is powerful because it makes no distributional assumptions about the placement qualities, and it directly learns the dual variables by replacing the expectation by a sample average function. In §EC.5 of the online appendix, we theoretically analyze the performance of an algorithm that learns in a nonparametric fashion the underlying parameters of the policy.

6.2. Impact of AdX

This first experiment explores the potential benefits of introducing AdX by studying the impact of the trade-off parameter γ on both objectives, that is, the

quality of the impressions assigned to the advertisers, and the revenue from AdX.

The experiment was conducted as follows. First, we set up a grid on the trade-off parameter γ and set $\tau = 0$. Then, we solve the SAA version of the dual problem via the SDM and determine the optimal dual variables and tie-breaking rule. Finally, we simulate the resulting modified bid-price policy μ^B on another sample of 400,000 impressions. We report in Table EC.1 in §EC.4 of the online appendix, the dual objective value J^D , optimality gap, AdX revenue J_A^D and contract quality J_C^D from the DAP, and AdX revenue J_A^B and contract quality J_C^B from simulating the bid-price policy for different choices of γ and different publishers. The objectives are normalized by dividing by the number of impressions in the horizon. In Figure 5 we plot, in a revenue versus quality graph, the performance of the

Table 3 Comparison of the Yields of the Optimal Policy with Those of the Greedy and Static-Price Policies for Publisher 3 and Different Choices of γ

Policy	γ	0.001	0.01	0.05	0.1	0.25	0.5	1	2.5	5	10	100
Bid-price Greedy	Yield ($J^{B(\gamma)}$)	1,829.4	1,834.9	1,859.2	1,887.9	1,994.2	2,193.7	2,614.6	3,951.1	6,283.9	11,060.4	98,570.7
	Yield	1,443.1	1,446.6	1,485.4	1,536.7	1,706.8	1,896.2	2,228.1	3,186.2	4,727.9	7,820.8	64,573.5
	Gap (%)	-21.12	-21.16	-20.11	-18.60	-14.41	-13.56	-14.78	-19.36	-24.76	-29.29	-34.49
Static-price	Yield	1,459.8	1,468.0	1,504.6	1,550.4	1,687.8	1,916.8	2,374.7	3,747.4	6,057.4	10,857.0	98,527.3
	Gap (%)	-20.21	-19.99	-19.07	-17.88	-15.36	-12.62	-9.18	-5.15	-3.60	-1.84	-0.04

DAP (J_A^D, J_C^D) and the simulated policy (J_A^B, J_C^B) for the different choices of γ .

Discussion. Results confirm that the efficient frontier achieved by the proposed heuristic μ^B is very close in relative terms to that of the DAP, and the proposed policy is approximately optimal for the stochastic control problem as claimed by Theorem 2. Additionally, the tie-breaking rule plays a significant role in the performance of the policy. Running the same experiments without the tie-breaking rule, we find that the publisher can incur losses in yield of up to 20% by failing to take ties into account.

In Figure 5 we observe that increasing the trade-off parameter γ increases the quality of the impressions assigned to the advertisers, and decreases the revenue from AdX. Interestingly, starting from the baseline case that disregards AdX ($\gamma = \infty$), we observe that the revenue from AdX can be substantially increased by sacrificing a small fraction of the overall quality of the impressions assigned. For instance, by strategically exploiting the AdX, publisher 1 can increase AdX revenue by 8% by giving up only 1% quality. Conversely, starting from the case that disregards the advertiser’s quality ($\gamma = 0$), the publisher can raise placement quality by a large amount at the expense of a small decrease in AdX revenue. Alternatively, the previous analysis can be understood in terms of the Pareto frontier. The Pareto frontier is highly concave, relatively horizontally flat around $\gamma = \infty$, and vertically flat around $\gamma = 0$. This explains the huge marginal improvements at the extremes.

6.3. Comparison with Greedy and Static-Price Policies

This second experiment compares the performance of our policy with the following heuristics.

- *The greedy policy* disregards the opportunity cost of capacity and assigns the impression to the advertiser with maximum quality. The policy is allowed to dynamically price and test the exchange before the assignment. Similar corrections to the ones in the original stochastic policy are introduced to guarantee that all contracts are satisfied almost surely. Note that the greedy policy is equivalent to setting the dual variables to $v_a = 0$ in the bid-price policy.
- *The static-price policy* prices in the exchange using the optimal reserve $p^*(0)$ throughout the horizon. The

policy is allowed to adjust the contracts’ qualities by choosing optimal bid prices and to assign rejected impressions to the contracts. In this case the publisher tests the exchange only if the marginal contribution of the exchange is positive or alternatively if $r(s^*(0)) \geq s^*(0)c$, where c denotes the maximum contract-adjusted quality.

Table 3 compares, by simulation, the yield of the optimal policy with the yield of the greedy and static-price policies for publisher 3 and different choices of γ .⁷

Discussion. Results confirm that the greedy policy underperforms in the given instances with losses in yield of up to 70%. From a managerial perspective, the suboptimality of the greedy policy stresses the importance of pondering the opportunity cost of capacity in performing the assignment of the impressions to the guaranteed contracts. If the publisher fails to take into account the opportunity cost of capacity, then some contracts are fulfilled early in the horizon and the opportunity to assign the top impressions is missed.

The static-price policy tends to underperform when the trade-off parameter γ is close to zero, that is, when the publisher strives to maximize the revenue extracted from AdX. By failing to dynamically adjust the auctions’ reserve price to take into account the opportunity cost of the impression, the publisher can incur losses in yield of up to 69%. From a managerial perspective, these results show that the ability to dynamically price plays a key role in the joint optimization between the guaranteed contracts and the spot market. When the trade-off parameter γ is large, the exchange’s revenue contribution to the yield is negligible, and the static-price policy is nearly optimal.

7. Extensions and Conclusion

Ad Exchanges are an emerging market for the real-time sale of online ad slots on the Internet. In this work we present an approach to help publishers to determine when and how to access AdX to complement their contract sales of impressions. In particular, we model

⁷ Because of space considerations, the results for the remaining publisher are available in SEC.4 of the online appendix. Results presented here are representative of the other publishers.

the publishers' problem as a stochastic control program and derive an asymptotically optimal policy with a simple structure: a bid-price control extended with a pricing function for the exchange. We show, using data from real inventory, that there are considerable advantages for publishers from joint optimization over both channels. Publishers may increase their revenue streams without giving away the quality of service of their reservation contracts, which still represent a significant portion of their advertising yield. We also hope that our insights here will further the understanding of ad allocation problems.

In §EC.1 of the online appendix, we consider a number of extensions to our model. First, we extend our model to account for the presence of multiple buyers in AdX. When the slot is sold via a second-price auction with reserve price, we provide conditions under which the revenue function remains regular and argue that all our results carry over to this setting. Second, we accommodate for covering constraints, that is, the case where the number of impressions assigned to each contract should be greater or equal to the capacity. These allow the publisher to exceed contractual targets in view of attracting future business, at the expense of reducing the revenue from the exchange. Third, we consider an alternative formulation in which the publisher specifies target quality constraints and maximizes AdX revenues, instead of maximizing a weighted combination of the objectives. Our method can be employed by interpreting the trade-off parameter γ as the Lagrange multiplier of the quality constraints.

Internet advertising, in particular AdX, is likely to prove to be a fertile area of research. There are several promising directions of research stemming from this work. One intuitive approach to improve the performance of a control, which is appealing for its simplicity, consists of resolving the deterministic approximation periodically throughout the horizon. Another problem that needs further study is that of learning in the case of unknown distributions, which is of great importance given the fast-paced and changing nature of the Internet. There exists independent research on online algorithms for capacity allocation and online pricing for repeated auctions, but none on the joint optimization problem. Finally, as more publishers reach out for AdX, advertisers will have the opportunity to buy their inventory from either market. The existence of two competing channels, the exchange as a spot market and the reservations as a future market, introduces several interesting research questions. How should publishers price their contracts and allocate their inventory? How should advertisers hedge their campaign between these two markets? We hope that this work paves the way for further research on this important topic.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2014.2017>.

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Appendix. Proofs of Selected Statements

PROOF OF PROPOSITION 1. In this proof we drop the dependence on the user attributes to simplify the notation. First, observe that for all c the objective function of (1) is concave and continuous in s , and the feasible set is compact. Hence, by Weierstrass theorem the set of optimal solutions is nonempty and compact. Thus, both $R(c)$ and $s^*(c)$ are well defined.

Second, $R(c) \geq c$ follows from letting $s = 0$. To see that $R(c)$ is nonincreasing, let $c < c'$, and let s^* be the optimal solution under cost c . Then, $R(c) = r(s^*) + (1 - s^*)c \leq r(s^*) + (1 - s^*)c' \leq R(c')$ where the first inequality follows because $s^* \leq 1$, and the second follows because no solution is better than the optimal. To see that $R(c) - c$ is nonincreasing, let $c' < c$, and let s^* be the optimal solution under cost c . Then, by a similar argument we get that $R(c') - c' \geq r(s^*) - s^*c' \geq r(s^*) - s^*c = R(c) - c$. Convexity follows in a similar way (this is a standard result).

Third, observe that the objective function of (1) is jointly continuous in s and c . Thus, by the maximum theorem $R(c)$ is continuous in c , and $s^*(c)$ is upper-hemicontinuous.

Finally, because $r(s) + (1 - s)c$ has decreasing differences in (s, c) and the feasible set is a lattice, by Topkis's theorem $s^*(c)$ is nonincreasing in c . The result for $p^*(c)$ follows from the fact that $\bar{F}^{-1}(s)$ is nonincreasing in s . \square

PROOF OF THEOREM 1. The optimality conditions of v for problem (4) imply that the directional derivative of $\psi(v)$ along any direction is greater than or equal to zero. In particular, for each advertiser $a \in \mathcal{A}$, it should be the case that $\nabla_{1_a} \psi(v) \geq 0$, and $\nabla_{-1_a} \psi(v) \geq 0$. Applying Proposition EC.2 of the online appendix to both directions, together with the fact that there is zero probability of a tie occurring, we get that

$$\mathbb{E}[(1 - s^*(\gamma Q_a - v_a); U) \mathbf{1}\{\gamma Q_a - v_a > \gamma Q_{a'} - v_{a'} \mid \forall a' \in \mathcal{A}_0\}] = \rho_a,$$

and the result follows. \square

PROOF OF PROPOSITION 2. The proof proceeds by contradiction: we first cast the feasible flow problem as a maximum flow problem, and then assume that there is no feasible flow. Feasibility would imply the existence of a flow with value $1 - \mathbb{P}(\emptyset\text{-tie})$ where $\mathbb{P}(\emptyset\text{-tie}) = \mathbb{E}[s^*(\max_{a \in \mathcal{A}_0} \{\gamma Q_a - v_a\}; U)]$. However, since we assume that no such feasible flow exists, by the max-flow min-cut theorem there should exist a cut with value strictly less than $1 - \mathbb{P}(\emptyset\text{-tie})$. The contradiction arises because the optimality conditions of v for the dual problem (4) imply that the every cut is lower-bounded by $1 - \mathbb{P}(\emptyset\text{-tie})$.

Step 1. We first cast the problem of finding a feasible tie-breaking probability as a flow problem. Let $y_a(S) = \mathbb{P}(S\text{-tie})p_a(S)$ be the total probability, originating from S -ties, of an impression being assigned to advertiser a . We can interpret $y_a(S)$ as the normalized flow of impressions assigned to the advertiser originating from S -ties. In terms of $y_a(S)$ as decision variables, finding the tie-breaking rule amounts to solving the transportation problem

$$\sum_{S \subseteq \mathcal{A}_0: a \in S} y_a(S) = \rho_a, \quad \forall a \in \mathcal{A}, \quad (8a)$$

$$\sum_{S \subseteq \mathcal{A}_0: 0 \in S} y_0(S) = \rho_0^{\text{eff}}, \quad (8b)$$

$$\sum_{a \in S} y_a(S) = \mathbb{P}(S\text{-tie}), \quad \forall S \subseteq \mathcal{A}_0, \quad (8c)$$

$$y_a(S) \geq 0, \quad \forall S \subseteq \mathcal{A}_0, a \in S. \quad (8d)$$

The constraints (8a) guarantee that for an advertiser $a \in \mathcal{A}$ the incoming flow of impressions over all possible ties sums up to ρ_a . Constraint (8b) imposes that the impressions effectively discarded are those that are rejected by AdX and not assigned to an advertiser, where we set $\rho_0^{\text{eff}} = 1 - \mathbb{P}(\emptyset\text{-tie}) - \sum_{a \in \mathcal{A}} \rho_a$. The constraints (8c) guarantee that the outgoing flow of impressions originating from a particular tie should sum up to the actual probability of that tie occurring.

The previous problem can be stated as a feasible flow problem in a bipartite graph. We briefly describe how to construct such a graph next. On the left-hand side of the graph we include one node for each nonempty subset $S \subseteq \mathcal{A}_0$ (the subset nodes), and on the right-hand side we add one node for each advertiser $a \in \mathcal{A}_0$ (the advertiser nodes). The supply for subset nodes is $\mathbb{P}(S\text{-tie})$, and the demand for advertiser nodes is ρ_a . Arcs in the graph represent the membership relation; i.e., the subset node S and advertiser node a are connected if and only if $a \in S$. Figure A.1(a) shows the resulting bipartite graph.

To write the feasible flow problem as a maximum flow problem, we first add a source s and a sink t . Second, we add one arc from s to each node associated with a nonempty subset $S \subseteq \mathcal{A}_0$ (left-hand side nodes) with capacity $\mathbb{P}(S\text{-tie})$. Third, we add one arc from each advertiser $a \in \mathcal{A}_0$ (right-hand side nodes) to t with capacity ρ_a . Lastly, we set the capacity of arcs from S to $a \in S$ to infinity.

Step 2. Now, since no feasible flow exists, by the max-flow min-cut theorem there should be a cut with a value strictly less than $1 - \mathbb{P}(\emptyset\text{-tie})$. Let $\alpha \subseteq \mathcal{A}_0$ be the advertiser nodes (right-hand side) belonging to the t side of a minimum cut. Figure A.1(b) shows the minimum cut. Next we argue that subset nodes on the s side verify that $S \cap \alpha = \emptyset$, whereas those on the t side verify that $S \cap \alpha \neq \emptyset$. First, because the cut has minimum value, there is no arc from a subset node to an advertiser node crossing the cut (those arcs have infinite capacity). Equivalently, on the s side of the cut, all subset nodes $S \subseteq \mathcal{A}_0$ should verify that $S \cap \alpha = \emptyset$. Second, observe that any subset node with $S \cap \alpha = \emptyset$ on the t side of the cut could be moved to the s side of the cut without increasing the value of the cut. Hence, with no loss of generality, we can assume that all subset nodes on the t side of the cut verify that $S \cap \alpha \neq \emptyset$.

As a consequence, the only arcs crossing the cut are those from the source to the subsets $S \cap \alpha \neq \emptyset$, and those from advertisers $\mathcal{A}_0 \setminus \alpha$ to the sink. The value of this cut is $\sum_{S \subseteq \mathcal{A}_0: S \cap \alpha \neq \emptyset} \mathbb{P}(S\text{-tie}) + \sum_{a \in \mathcal{A}_0 \setminus \alpha} \rho_a$. Because the value is strictly less than $1 - \mathbb{P}(\emptyset\text{-tie})$, we get that

$$\sum_{S \subseteq \mathcal{A}_0: S \cap \alpha \neq \emptyset} \mathbb{P}(S\text{-tie}) < \sum_{a \in \alpha} \rho_a, \quad (9)$$

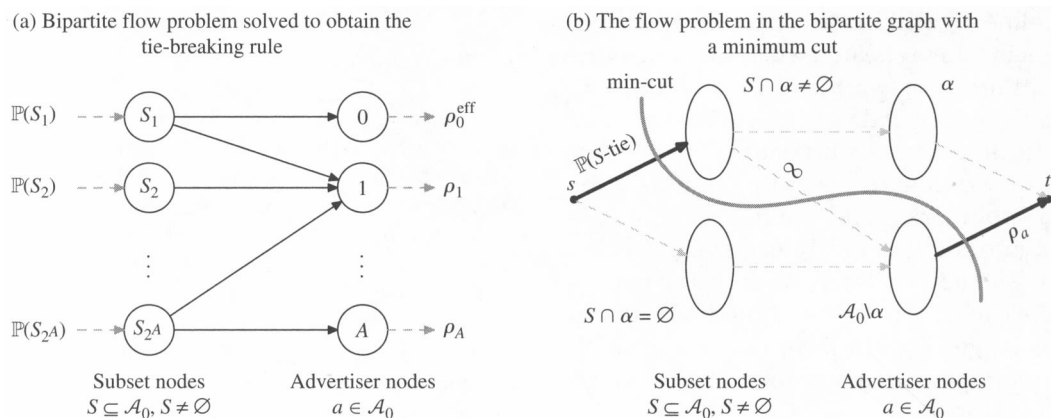
where we use that $\sum_{a \in \mathcal{A}} \rho_a + \rho_0^{\text{eff}} = 1 - \mathbb{P}(\emptyset\text{-tie})$.

Next, we look at the optimality conditions of v for the dual problem (4). We distinguish between the case that $0 \notin \alpha$ and $0 \in \alpha$. First suppose that $0 \notin \alpha$, and consider the direction $-\mathbf{1}_\alpha$ that has a -1 if $a \in \alpha$ and 0 elsewhere. According to Proposition EC.2 of the online appendix, the directional derivative of the normalized dual objective $\bar{\psi}(v) = \psi(v)/N$ at v is

$$\begin{aligned} \nabla_{-\mathbf{1}_\alpha} \bar{\psi}(v) &= \mathbb{P}_R \left\{ \max_{a \in \alpha} \{\gamma Q_a - v_a\} \geq \max_{a \in \mathcal{A}_0 \setminus \alpha} \{\gamma Q_a - v_a\} \right\} - \sum_{a \in \alpha} \rho_a \\ &= \sum_{S \subseteq \mathcal{A}_0: S \cap \alpha \neq \emptyset} \mathbb{P}(S\text{-tie}) - \sum_{a \in \alpha} \rho_a, \end{aligned}$$

where we have written the event that the maximum is verified nonexclusively by some advertiser $a \in \alpha$ as all S -ties in which some advertiser $a \in \alpha$ is involved. The optimality of v implies that the directional derivative along that direction is greater than or equal to zero, contradicting Equation (9).

Figure A.1 Graphical Representation of the Bipartite Flow Problem Solved to Obtain the Tie-Breaking Rule



When $0 \in \alpha$ we consider the direction $\mathbf{1}_{\mathcal{A} \setminus \alpha}$ that has a 1 if $a \notin \alpha$ and 0 elsewhere. The direction derivative is now

$$\begin{aligned} \nabla_{\mathbf{1}_{\mathcal{A} \setminus \alpha}} \bar{\psi}(v) &= -\mathbb{P}_R \left\{ \max_{a \in \mathcal{A} \setminus \alpha} \{\gamma Q_a - v_a\} > \max_{a \in \alpha \cup \{0\}} \{\gamma Q_a - v_a\} \right\} \\ &\quad + \sum_{a \in \mathcal{A} \setminus \alpha} \rho_a, \\ &= - \sum_{S \subseteq \mathcal{A}_0: S \subseteq \mathcal{A} \setminus \alpha} \mathbb{P}(S\text{-tie}) + \sum_{a \in \mathcal{A} \setminus \alpha} \rho_a \\ &= \sum_{S \subseteq \mathcal{A}_0: S \cap \alpha \neq \emptyset} \mathbb{P}(S\text{-tie}) - \sum_{a \in \alpha} \rho_a, \end{aligned}$$

where in the second equation we have written the event that the maximum is verified exclusively by some advertiser $a \in \alpha$ as all S -ties in which only advertisers in α are involved. The third equation follows because $\mathcal{A} \setminus \alpha = \mathcal{A}_0 \setminus \alpha$ since $0 \in \alpha$ together with the fact that $\mathbb{P}(\emptyset\text{-tie}) + \sum_{S \subseteq \mathcal{A}_0: S \cap \alpha \neq \emptyset} \mathbb{P}(S\text{-tie}) + \sum_{S \subseteq \mathcal{A}_0 \setminus \alpha} \mathbb{P}(S\text{-tie}) = 1$ and that $\sum_{a \in \mathcal{A} \setminus \alpha} \rho_a + \sum_{a \in \alpha} \rho_a = 1 - \mathbb{P}(\emptyset\text{-tie})$. Again, the optimality of v implies that the directional derivative along that direction is greater than or equal to zero, contradicting Equation (9). \square

PROOF OF PROPOSITION 3. First, we formally define the set of stochastic control policies. Second, we proceed by taking any feasible stochastic control policy and constructing a feasible solution for the DAP by taking expectations over the history. Exploiting concavity, we show that the performance of this new solution Pareto dominates that of the original control in the stochastic framework. We conclude by showing that the efficient frontier of the DAP is concave, and prove that any Pareto point can be achieved by optimizing a weighted combination of the objectives.

Step 1. A stochastic control policy maps states of the system to control actions (prices and target advertiser), and is adapted to the history up to the decision epoch. We restrict our attention to policies that always submit the impression to AdX, which were argued to be optimal. As before, we recast the problem in terms of the survival probability control, and the publisher picks the probability that the impression is accepted. We denote by $s_n^\mu(U) \in [0, 1]$ the target survival probability under policy μ at time n when an impression with attributes u arrives. Similarly, we let $I_{n,a}^\mu(u) \in \{0, 1\}$ indicate whether the n th impression is assigned to advertiser a or not when policy μ is used. In particular, $I_{n,a}^\mu(u) = 1$ indicates that the impression should be assigned to the advertiser if rejected by AdX.

We let the binary random variable $X_n(s_n^\mu)$ indicate whether the n th impression is accepted by AdX or not when policy μ is used. Specifically, $X_n(s_n^\mu) = 1$ indicates that the impression is accepted by AdX, and $X_n(s_n^\mu) = 0$ indicates that the impression is rejected by AdX. Note that, conditioning on impression's attributes and the history, $X_n(s_n^\mu)$ is a Bernoulli random variable with success probability s_n^μ .

We denote by \mathcal{M} the set of admissible policies, i.e., policies that are nonanticipating, adapting, and feasible. A feasible policy should satisfy the contractual obligations with each advertiser, or equivalently $\sum_{n=1}^N [1 - X_n(s_n^\mu)] I_{n,a}^\mu = C_a$ in an almost sure sense. Additionally, the target advertiser controls should satisfy that $\sum_{a \in \mathcal{A}} I_{n,a}^\mu \leq 1$, since the impression should be assigned to at most one advertiser.

We denote by $J_A^\mu = \mathbb{E}[\sum_{n=1}^N r(s_n^\mu(U_n); U_n)]$ the expected AdX revenue and by $J_C^\mu = \mathbb{E}[(1 - s_n^\mu(U_n)) \sum_{a \in \mathcal{A}} I_{n,a}^\mu(U_n) Q_{n,a}]$ the expected contract quality for policy μ .

Step 2. Next we show that for any admissible policy $\mu \in \mathcal{M}$ there exists a vector of deterministic controls that is feasible for the DAP, and its performance on the DAP Pareto dominates its performance in the stochastic framework. Let $\hat{s} = \{\hat{s}_n(\cdot)\}_{n=1}^N$ and $\hat{i} = \{\hat{i}_n(\cdot)\}_{n=1}^N$ be deterministic vectors of controls defined as

$$\hat{s}_n(u) = \mathbb{E}_{\mathcal{F}_n}[s_n^\mu(u) | u] \quad \forall u \text{ pointwise,}$$

$$\hat{i}_{n,a}(u) = \mathbb{E}_{\mathcal{F}_n}[(1 - s_n^\mu(u)) I_{n,a}^\mu(u) | u] \quad \forall u \text{ pointwise, } a \in \mathcal{A},$$

where the expectation is taken over the history of the system until n , which is denoted by \mathcal{F}_n , and is conditional on a particular realization of u . The resulting controls are independent of the history, and dependent only on the realization of u and the impression number n . Thus, they fulfill the first approximation and are valid deterministic vectors of controls.

First, for the contract fulfillment constraint we have that for each advertiser $a \in \mathcal{A}$

$$\begin{aligned} C_a &= \mathbb{E} \left[\sum_{n=1}^N (1 - X_n(s_n^\mu(U_n))) I_{n,a}^\mu(U_n) \right] \\ &= \sum_{n=1}^N \mathbb{E}[\mathbb{E}_{\mathcal{F}_n}[(1 - s_n^\mu(U_n)) I_{n,a}^\mu(U_n) | U_n]] = \sum_{n=1}^N \mathbb{E}[\hat{i}_{n,a}(U)], \end{aligned}$$

where the first equality follows from taking expectations to the almost sure contract fulfillment constraint of μ , the second from the tower rule, and the third from substituting \hat{s} and \hat{i} pointwise for all U and the fact that impressions are i.i.d. Nonnegativity of the controls follows trivially. Additionally, is it not hard to show that $\sum_{a \in \mathcal{A}} \hat{i}_{n,a}(\cdot) + s_n(\cdot) \leq 1$ for all n . Thus, (\hat{s}, \hat{i}) is a feasible deterministic control.

Second, let $J_A^D(\hat{s}, \hat{i}) = \mathbb{E}[\sum_{n=1}^N r(\hat{s}_n(U_n); U_n)]$ be the expected AdX revenue and $J_C^D(\hat{s}, \hat{i}) = \mathbb{E}[\sum_{a \in \mathcal{A}} Q_{n,a} \hat{i}_{n,a}(U_n)]$ the expected contract quality of the deterministic control (\hat{s}, \hat{i}) . In terms of the revenue we have that

$$\begin{aligned} J_A^\mu &= \sum_{n=1}^N \mathbb{E}[r(s_n^\mu(U_n); U_n)] = \sum_{n=1}^N \mathbb{E}[\mathbb{E}_{\mathcal{F}_n}[r(s_n^\mu(U_n); U_n) | U_n]] \\ &\leq \sum_{n=1}^N \mathbb{E}[r(\hat{s}_n(U_n); U_n)] = J_A^D(\hat{s}, \hat{i}), \end{aligned}$$

where the second equality follows from the tower rule and because U_n is measurable w.r.t. the conditional expectation, and the inequality follows from applying Jensen's inequality to the concave revenue function. Similarly for the quality we have that

$$\begin{aligned} J_C^\mu &= \sum_{n=1}^N \mathbb{E} \left[(1 - s_n^\mu(U_n)) \sum_{a \in \mathcal{A}} I_{n,a}^\mu(U_n) Q_{n,a} \right] \\ &= \sum_{n=1}^N \mathbb{E} \left[\sum_{a \in \mathcal{A}} Q_{n,a} \mathbb{E}_{\mathcal{F}_n}[(1 - s_n^\mu(U_n)) I_{n,a}^\mu(U_n) | U_n] \right] \\ &= \mathbb{E} \left[\sum_{a \in \mathcal{A}} Q_{n,a} \hat{i}_{n,a}(U_n) \right] = J_C^D(\hat{s}, \hat{i}). \end{aligned}$$

Step 3. Letting (\vec{s}^*, \vec{i}^*) be a Pareto-optimal point for the DAP, we would like to show that there exists a trade-off parameter γ such that this point is the optimal solution of

the γ -weighted DAP. Let $\mathcal{C} = \{(j_A, j_C) \in \mathbb{R}^2: j_A \leq J_A(\vec{s}, \vec{i}), j_C \leq J_C(\vec{s}, \vec{i}) \text{ for some DAP-feasible } (\vec{s}, \vec{i})\}$ be the set of all values that are worse than or equal to some achievable objective value for the DAP. We first show that the set \mathcal{C} is convex by proving that for any $(j_A^1, j_C^1), (j_A^2, j_C^2) \in \mathcal{C}$ and $\lambda \in (0, 1)$, the convex combination $\lambda(j_A^1, j_C^1) + (1 - \lambda)(j_A^2, j_C^2)$ lies in the set \mathcal{C} .

Note that there exist DAP-feasible controls (\vec{s}^k, \vec{i}^k) for $k = 1, 2$ such that $j_A^k \leq J_A(\vec{s}^k, \vec{i}^k)$ and $j_C^k \leq J_C(\vec{s}^k, \vec{i}^k)$. Consider the convex combination of the controls (\vec{s}, \vec{i}) given by $s_n(\cdot) = \lambda s_n^1(\cdot) + (1 - \lambda)s_n^2(\cdot)$ and $i_{n,a}(\cdot) = \lambda i_{n,a}^1(\cdot) + (1 - \lambda)i_{n,a}^2(\cdot)$ for every $n = 1, \dots, N$ and $a \in \mathcal{A}$. The convex combination is DAP-feasible; that is, (i) $(s_n, i_n) \in \mathcal{P}$ because the probability simplex \mathcal{P} is convex, and (ii) $\sum_{n=1}^N \mathbb{E}[i_{n,a}] = C_a$ for every $a \in \mathcal{A}$ because the capacity constraints and the expectation operator are linear. For the AdX revenue we have that

$$\begin{aligned} \lambda j_A^1 + (1 - \lambda)j_A^2 &\leq \lambda J_A(\vec{s}^1, \vec{i}^1) + (1 - \lambda)J_A(\vec{s}^2, \vec{i}^2) \\ &= \sum_{n=1}^N \mathbb{E}[\lambda r(\hat{s}_n^1(U_n); U_n) + (1 - \lambda)r(\hat{s}_n^2(U_n); U_n)] \\ &\leq \sum_{n=1}^N \mathbb{E}[r(\lambda \hat{s}_n^1(U_n) + (1 - \lambda)\hat{s}_n^2(U_n); U_n)] = J_A(\vec{s}, \vec{i}), \end{aligned}$$

where the second equality follows from Jensen's inequality and the concavity of the revenue function. A similar argument follows for the contract quality and thus the convexity of the set \mathcal{C} holds.

Clearly, the Pareto-optimal point (\vec{s}^*, \vec{i}^*) lies at the boundary of the set \mathcal{C} . Because the set \mathcal{C} is convex and nonempty, by the supporting hyperplane theorem there exists a vector of weights $(\gamma_A, \gamma_C) \neq 0$ such that $\gamma_A J_A(\vec{s}^*, \vec{i}^*) + \gamma_C J_C(\vec{s}^*, \vec{i}^*) \geq \gamma_A J_A(\vec{s}, \vec{i}) + \gamma_C J_C(\vec{s}, \vec{i})$ for every DAP-feasible control (\vec{s}, \vec{i}) . Because the set \mathcal{C} is unbounded from below and from the left, we conclude that $(\gamma_A, \gamma_C) \geq 0$. The result follows from setting $\gamma = \gamma_C / \gamma_A$. The extreme Pareto points corresponding to $\gamma = 0, \infty$ can be achieved as the limits of Pareto points of positive parameters (see, e.g., Boyd and Vandenberghe 2009).

PROOF OF THEOREM 2. We prove the bound for the yield of the policy. The bounds for the AdX revenue and contract quality follow mutatis mutandis. In the remainder of the proof, time periods are indexed forward in time, and we drop the dependence on the fixed trade-off parameter γ .

Let $S_{n,a}^\mu = \sum_{i=1}^n (1 - X_i(s_i^\mu(U_i)))I_{i,a}^\mu(U_i)$ be the total number of impressions assigned to advertiser a by time n when following the stochastic policy μ^B . Additionally, we denote by $S_n^\mu = \{S_{n,a}^\mu\}_{a \in \mathcal{A}}$ the random vector of impressions assigned to advertisers.

To simplify the proof, we let $C_0 = N - \sum_{a \in \mathcal{A}} C_a$ be the total number of impressions that are not assigned to any advertiser (accepted by AdX and discarded), and we refer to $S_{n,0}^\mu = n - \sum_{a \in \mathcal{A}} S_{n,a}^\mu$ as the total number of impressions not assigned to any advertiser by time n when following the stochastic policy μ^B . Because C_0 is the total number of impressions of which we can dispense, when the point is reached that $S_{n,0} = C_0$, then all remaining impressions need to be assigned to the advertisers.

Let the random time $N^* = \inf\{1 \leq n \leq N: S_{n,a}^\mu = C_a \text{ for some } a \in \mathcal{A}\}$ be the first time that any advertiser's contract is fulfilled or the point is reached where all arriving impressions need to be assigned to the advertisers. Clearly, N^* is a stopping time with respect to the stochastic process

$\{S_n^\mu\}_{n=1}^N$. In the following, let Y_n^μ be the yield from impression n under policy μ^B . Similarly, we denote by Y_n the yield from impression n when the deterministic controls are used in an alternate coupled system with no capacity constraints. Because the deterministic controls are time homogeneous, and the underlying random variables are i.i.d., then the random variables $\{Y_n\}_{n=1}^N$ are i.i.d. too. Note that when $n < N^*$, the controls of the stochastic policy μ^B behave exactly as the optimal deterministic controls. Thus, $Y_n = Y_n^\mu$ for $n < N^*$. Using this fact together with the fact that N^* is a stopping time we get that

$$\begin{aligned} J^B &= \mathbb{E}\left[\sum_{n=1}^N Y_n^\mu\right] = \mathbb{E}\left[\sum_{n=1}^{N^*} Y_n + \sum_{n=N^*+1}^N Y_n^\mu\right] \geq \mathbb{E}\left[\sum_{n=1}^{N^*} Y_n\right] \\ &= \mathbb{E}[N^*] \mathbb{E}[Y_n], \end{aligned}$$

where the inequality follows from the nonnegativity of yield, and the last equality follows from Wald's equation. Then, we conclude that $J^B/J^D \geq \mathbb{E}N^*/N$ because $N \mathbb{E}Y_n = J^D$.

Next, we turn to the problem of providing a lower bound for $\mathbb{E}N^*$. Before proceeding we make some definitions. We define by $S_{n,a}$ the number of impressions assigned to advertiser a by time n when following the deterministic controls in the alternate system with no capacity constraints. As for the yield, it is the case that $S_{n,a} = S_{n,a}^\mu$ for $n < N^*$. We define $S_{n,0}$ in a similar fashion.

Let $N_a = \inf\{n \geq 1: S_{n,a} = C_a\}$ be the time when the contract of advertiser $a \in \mathcal{A}$ is fulfilled, and $N_0 = \inf\{n \geq 1: S_{n,0} = C_0\}$ be the point in time where all arriving impressions need to be assigned to the advertisers. Although these stopping times are defined with respect to the stochastic process that follows the deterministic controls, it is the case that $N^* \stackrel{(d)}{=} \min_{a \in \mathcal{A}_0} \{N_a\}$. In the remainder of the proof we study the mean and variance of each stopping time and then conclude with a bound for $\mathbb{E}N^*$ based on those central moments.

For the case of $a \in \mathcal{A}$, the summands of $S_{n,a}$ are independent Bernoulli random variables with success probability ρ_a . The success probability follows from (3). Hence, $N_a - C_a$ is distributed as a negative binomial random variable with C_a successes and success probability ρ_a . The mean and variance are given by $\mathbb{E}N_a = N$, and $\text{Var}[N_a] = N((1 - \rho_a)/\rho_a)$, where we use that $\rho_a = C_a/N$. Similarly, for the case of $a = 0$, now the summands of $S_{n,0}$ are Bernoulli random variables with success probability ρ_0 . Hence, $N_0 - C_0$ is distributed as a negative binomial random variable with C_0 successes and success probability ρ_0 .

Finally, using the lower bound on the mean of the minimum of a number of random variables of Aven (1985) we get that

$$\begin{aligned} \mathbb{E}N^* &= \mathbb{E}\min_{a \in \mathcal{A}_0} \{N_a\} \geq \min_{a \in \mathcal{A}_0} \mathbb{E}N_a - \sqrt{\frac{A}{A+1} \sum_{a \in \mathcal{A}_0} \text{Var}[N_a]} \\ &= N - \sqrt{\frac{A}{A+1} \sum_{a \in \mathcal{A}_0} N \frac{1 - \rho_a}{\rho_a}} = N - \sqrt{NK(\rho)}, \end{aligned}$$

and the result follows. \square

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EXHIBIT E

GOOGLE PRIVACY POLICY

When you use our services, you're trusting us with your information. We understand this is a big responsibility and work hard to protect your information and put you in control.

This Privacy Policy is meant to help you understand what information we collect, why we collect it, and how you can update, manage, export, and delete your information.

Effective December 19, 2019 | [Archived versions](#) | [Download PDF](#)

We build a range of services that help millions of people daily to explore and interact with the world in new ways. Our services include:

- Google apps, sites, and devices, like Search, YouTube, and Google Home
- Platforms like the Chrome browser and Android operating system
- Products that are integrated into third-party apps and sites, like ads and embedded Google Maps

You can use our services in a variety of ways to manage your privacy. For example, you can sign up for a Google Account if you want to create and manage content like emails and photos, or see more relevant search results. And you can use many Google services when you're signed out or without creating an account at all, like searching on Google or watching YouTube videos. You can also choose to browse the web privately using Chrome in Incognito mode. And across our services, you can adjust your privacy settings to control what we collect and how your information is used.

To help explain things as clearly as possible, we've added examples, explanatory videos, and definitions for [key terms](#). And if you have any questions about this Privacy Policy, you can [contact us](#).

INFORMATION GOOGLE COLLECTS

We want you to understand the types of information we collect as you use our services

We collect information to provide better services to all our users — from figuring out basic stuff like which language you speak, to more complex things like which [ads you'll find most useful](#), [the people who matter most to you online](#), or which YouTube videos you might like. The information Google collects, and how that information is used, depends on how you use our services and how you manage your privacy controls.

When you're not signed in to a Google Account, we store the information we collect with [unique identifiers](#) tied to the browser, application, or [device](#) you're using. This helps us do things like maintain your language preferences across browsing sessions.

When you're signed in, we also collect information that we store with your Google Account, which we treat as [personal information](#).

Things you create or provide to us

When you create a Google Account, you provide us with [personal information](#) that includes your name and a password. You can also choose to add a [phone number](#) or [payment information](#) to your account. Even if you aren't signed in to a Google Account, you might choose to provide us with information — like an email address to receive updates about our services.

We also collect the content you create, upload, or receive from others when using our services. This includes things like email you write and receive, photos and videos you save, docs and spreadsheets you create, and comments you make on YouTube videos.

Information we collect as you use our services

Your apps, browsers & devices

We collect information about the apps, browsers, and [devices](#) you use to access Google services, which helps us provide features like automatic product updates and dimming your screen if your battery runs low.

The information we collect includes [unique identifiers](#), browser type and settings, device type and settings, operating system, mobile network information including carrier name and phone number, and application version number. We also collect information about the interaction of your apps, browsers, and devices with our services, including [IP address](#), crash reports, system activity, and the date, time, and referrer URL of your request.

We collect this information when a Google service on your device contacts our servers – for example, when you install an app from the Play Store or when a service checks for automatic updates. If you're using an [Android device with Google apps](#), your device periodically contacts Google servers to provide information about your device and connection to our services. This information includes things like your device type, carrier name, crash reports, and which apps you've installed.

Your activity

We collect information about your activity in our services, which we use to do things like recommend a YouTube video you might like. The activity information we collect may include:

- Terms you search for
- Videos you watch
- [Views and interactions with content and ads](#)
- Voice and audio information when you use audio features
- Purchase activity
- People with whom you communicate or share content
- Activity on third-party sites and apps that use our services
- Chrome browsing history you've [synced with your Google Account](#)

If you use our [services to make and receive calls or send and receive messages](#), we may collect telephony log information like your phone number, calling-party number, receiving-

party number, forwarding numbers, time and date of calls and messages, duration of calls, routing information, and types of calls.

You can visit your Google Account to find and manage activity information that's saved in your account.

[Go to Google Account](#)

Your location information

We collect information about your location when you use our services, which helps us offer features like driving directions for your weekend getaway or showtimes for movies playing near you.

Your location can be determined with varying degrees of accuracy by:

- GPS
- [IP address](#)
- [Sensor data from your device](#)
- [Information about things near your device](#), such as Wi-Fi access points, cell towers, and Bluetooth-enabled devices

The types of location data we collect depend in part on your device and account settings. For example, you can [turn your Android device's location on or off](#) using the device's settings app. You can also turn on [Location History](#) if you want to create a private map of where you go with your signed-in devices.

In some circumstances, Google also collects information about you from [publicly accessible sources](#). For example, if your name appears in your local newspaper, Google's Search engine may index that article and display it to other people if they search for your name. We may also collect information about you from trusted partners, including marketing partners who provide us with information about potential customers of our business services, and security partners who provide us with information to [protect](#)

against abuse. We also receive information from advertisers to provide advertising and research services on their behalf.

We use various technologies to collect and store information, including cookies, pixel tags, local storage, such as browser web storage or application data caches, databases, and server logs.

WHY GOOGLE COLLECTS DATA

We use data to build better services

We use the information we collect from all our services for the following purposes:

Provide our services

We use your information to deliver our services, like processing the terms you search for in order to return results or helping you share content by suggesting recipients from your contacts.

Maintain & improve our services

We also use your information to ensure our services are working as intended, such as tracking outages or troubleshooting issues that you report to us. And we use your information to make improvements to our services — for example, understanding which search terms are most frequently misspelled helps us improve spell-check features used across our services.

Develop new services

We use the information we collect in existing services to help us develop new ones. For example, understanding how people organized their photos in Picasa, Google's first photos app, helped us design and launch Google Photos.

Provide personalized services, including content and ads

We use the information we collect to customize our services for you, including providing recommendations, personalized content, and [customized search results](#). For example, [Security Checkup](#) provides security tips adapted to how you use Google products. And Google Play uses information like apps you've already installed and videos you've watched on YouTube to suggest new apps you might like.

Depending on your settings, we may also show you [personalized ads](#) based on your interests. For example, if you search for "mountain bikes," you may see an ad for sports equipment when you're browsing a site that shows ads served by Google. You can control what information we use to show you ads by visiting your ad settings.

- We don't show you personalized ads based on [sensitive categories](#), such as race, religion, sexual orientation, or health.
- We don't share information that personally identifies you with advertisers, such as your name or email, unless you ask us to. For example, if you see an ad for a nearby flower shop and select the "tap to call" button, we'll connect your call and may share your phone number with the flower shop.

[Go to Ad Settings](#)

Measure performance

We use data for analytics and measurement to understand how our services are used. For example, we analyze data about your visits to our sites to do things like optimize product design. And we also use data about the ads you interact with to help advertisers understand the performance of their ad campaigns. We use a variety of tools to do this, including Google Analytics. When you visit sites that use Google Analytics, Google and a Google Analytics customer [may link information](#) about your activity from that site with activity from other sites that use our ad services.

Communicate with you

We use information we collect, like your email address, to interact with you directly. For example, we may send you a notification if we detect suspicious activity, like an attempt to sign in to your Google Account from an unusual location. Or we may let you know

about upcoming changes or improvements to our services. And if you contact Google, we'll keep a record of your request in order to help solve any issues you might be facing.

Protect Google, our users, and the public

We use information to help improve the [safety and reliability](#) of our services. This includes detecting, preventing, and responding to fraud, abuse, security risks, and technical issues that could harm Google, our users, or the public.

We use different technologies to process your information for these purposes. We use automated systems that analyze your content to provide you with things like customized search results, personalized ads, or other features tailored to how you use our services. And we analyze your content to help us [detect abuse](#) such as spam, malware, and illegal content. We also use [algorithms](#) to recognize patterns in data. For example, Google Translate helps people communicate across languages by detecting common language patterns in phrases you ask it to translate.

We may [combine the information we collect](#) among our services and across your devices for the purposes described above. For example, if you watch videos of guitar players on YouTube, you might see an ad for guitar lessons on a site that uses our ad products. Depending on your account settings, [your activity on other sites and apps](#) may be associated with your personal information in order to improve Google's services and the ads delivered by Google.

If other users already have your email address or other information that identifies you, we may show them your publicly visible Google Account information, such as your name and photo. This helps people identify an email coming from you, for example.

We'll ask for your consent before using your information for a purpose that isn't covered in this Privacy Policy.

YOUR PRIVACY CONTROLS

You have choices regarding the information we collect and how it's used

This section describes key controls for managing your privacy across our services. You can also visit the [Privacy Checkup](#), which provides an opportunity to review and adjust important privacy settings. In addition to these tools, we also offer specific privacy settings in our products — you can learn more in our [Product Privacy Guide](#).

[Go to Privacy Checkup](#)

Managing, reviewing, and updating your information

When you're signed in, you can always review and update information by visiting the services you use. For example, Photos and Drive are both designed to help you manage specific types of content you've saved with Google.

We also built a place for you to review and control information saved in your Google Account. Your [Google Account](#) includes:

Privacy controls

Activity Controls

Decide what types of activity you'd like saved in your account. For example, you can turn on Location History if you want traffic predictions for your daily commute, or you can save your YouTube Watch History to get better video suggestions.

[Go to Activity Controls](#)

Ad settings

Manage your preferences about the ads shown to you on Google and on sites and apps that [partner with Google](#) to show ads. You can modify your interests, choose whether

your personal information is used to make ads more relevant to you, and turn on or off certain advertising services.

[Go to Ad Settings](#)

About you

Control what others see about you across Google services.

[Go to About You](#)

Shared endorsements

Choose whether your name and photo appear next to your activity, like reviews and recommendations, that appear in ads.

[Go to Shared Endorsements](#)

Information you share

If you're a G Suite user, control whom you share information with through your account on Google+.

[Go to Information You Share](#)

Ways to review & update your information

My Activity

My Activity allows you to review and control data that's created when you use Google services, like searches you've done or your visits to Google Play. You can browse by date and by topic, and delete part or all of your activity.

[Go to My Activity](#)

Google Dashboard

Google Dashboard allows you to manage information associated with specific products.

[Go to Dashboard](#)

Your personal information

Manage your contact information, such as your name, email, and phone number.

[Go to Personal Info](#)

When you're signed out, you can manage information associated with your browser or device, including:

- Signed-out search personalization: [Choose](#) whether your search activity is used to offer you more relevant results and recommendations.
- YouTube settings: Pause and delete your [YouTube Search History](#) and your [YouTube Watch History](#).
- Ad Settings: [Manage](#) your preferences about the ads shown to you on Google and on sites and apps that partner with Google to show ads.

Exporting, removing & deleting your information

You can export a copy of content in your Google Account if you want to back it up or use it with a service outside of Google.

[Export your data](#)

You can also [request to remove content](#) from specific Google services based on applicable law.

To delete your information, you can:

- Delete your content from [specific Google services](#)
- Search for and then delete specific items from your account using [My Activity](#)
- [Delete specific Google products](#), including your information associated with those products
- [Delete your entire Google Account](#)

[Delete your information](#)

And finally, [Inactive Account Manager](#) allows you to give someone else access to parts of your Google Account in case you're unexpectedly unable to use your account.

There are other ways to control the information Google collects whether or not you're signed in to a Google Account, including:

- **Browser settings:** For example, you can configure your browser to indicate when Google has set a [cookie](#) in your browser. You can also configure your browser to block all cookies from a specific domain or all domains. But remember that our services [rely on cookies to function properly](#), for things like remembering your language preferences.
- **Device-level settings:** Your device may have controls that determine what information we collect. For example, you can [modify location settings](#) on your Android device.

SHARING YOUR INFORMATION

When you share your information

Many of our services let you share information with other people, and you have control over how you share. For example, you can share videos on YouTube publicly or you can decide to keep your videos private. Remember, when you share information publicly, your content may become accessible through search engines, including Google Search.

When you're signed in and interact with some Google services, like leaving comments on a YouTube video or reviewing an app in Play, your name and photo appear next to your activity. We may also display this information in [ads depending on your Shared endorsements setting](#).

When Google shares your information

We do not share your personal information with companies, organizations, or individuals outside of Google except in the following cases:

With your consent

We'll share personal information outside of Google when we have your consent. For example, if you [use Google Home to make a reservation](#) through a booking service, we'll get your permission before sharing your name or phone number with the restaurant. We'll ask for your explicit consent to share any [sensitive personal information](#).

With domain administrators

If you're a student or work for an organization that uses Google services (like G Suite), your [domain administrator](#) and resellers who manage your account will have access to your Google Account. They may be able to:

- Access and retain information stored in your account, like your email
- View statistics regarding your account, like how many apps you install
- Change your account password

- Suspend or terminate your account access
- Receive your account information in order to satisfy applicable law, regulation, legal process, or enforceable governmental request
- Restrict your ability to delete or edit your information or your privacy settings

For external processing

We provide personal information to our [affiliates](#) and other trusted businesses or persons to process it for us, based on our instructions and in compliance with our Privacy Policy and any other appropriate confidentiality and security measures. For example, we use service providers to help us with customer support.

For legal reasons

We will share personal information outside of Google if we have a good-faith belief that access, use, preservation, or disclosure of the information is reasonably necessary to:

- Meet any applicable law, regulation, [legal process, or enforceable governmental request](#). We share information about the number and type of requests we receive from governments in our [Transparency Report](#).
- Enforce applicable Terms of Service, including investigation of potential violations.
- Detect, prevent, or otherwise address fraud, security, or technical issues.
- Protect against harm to the rights, property or safety of Google, our users, or the public as required or permitted by law.

We may share [non-personally identifiable information](#) publicly and with our partners — like publishers, advertisers, developers, or rights holders. For example, we share information publicly to [show trends](#) about the general use of our services. We also allow [specific partners](#) to collect information from your browser or device for advertising and measurement purposes using their own cookies or similar technologies.

If Google is involved in a merger, acquisition, or sale of assets, we'll continue to ensure the confidentiality of your personal information and give affected users notice before personal information is transferred or becomes subject to a different privacy policy.

KEEPING YOUR INFORMATION SECURE

We build security into our services to protect your information

All Google products are built with strong security features that continuously protect your information. The insights we gain from maintaining our services help us detect and automatically block security threats from ever reaching you. And if we do detect something risky that we think you should know about, we'll notify you and help guide you through steps to stay better protected.

We work hard to protect you and Google from unauthorized access, alteration, disclosure, or destruction of information we hold, including:

- We use encryption to keep your data private while in transit
- We offer a range of security features, like [Safe Browsing](#), Security Checkup, and [2 Step Verification](#) to help you protect your account
- We review our information collection, storage, and processing practices, including physical security measures, to prevent unauthorized access to our systems
- We restrict access to personal information to Google employees, contractors, and agents who need that information in order to process it. Anyone with this access is subject to strict contractual confidentiality obligations and may be disciplined or terminated if they fail to meet these obligations.

EXPORTING & DELETING YOUR INFORMATION

You can export a copy of your information or delete it from your Google Account at any time

You can export a copy of content in your Google Account if you want to back it up or use it with a service outside of Google.

[Export your data](#)

To delete your information, you can:

- Delete your content from [specific Google services](#)
- Search for and then delete specific items from your account using [My Activity](#)
- [Delete specific Google products](#), including your information associated with those products
- [Delete your entire Google Account](#)

[Delete your information](#)

RETAINING YOUR INFORMATION

We retain the data we collect for different periods of time depending on what it is, how we use it, and how you configure your settings:

- Some data you can delete whenever you like, such as the content you create or upload. You can also delete [activity information](#) saved in your account, or [choose to have it deleted automatically](#) after a set period of time.
- Other data is deleted or anonymized automatically after a set period of time, such as [advertising data](#) in server logs.
- We keep some data until you delete your Google Account, such as information about how often you use our services.
- And some data we retain for longer periods of time when necessary for legitimate business or legal purposes, such as security, fraud and abuse prevention, or financial record-keeping.

When you delete data, we follow a deletion process to make sure that your data is safely and completely removed from our servers or retained only in anonymized form. We try to ensure that our services protect information from accidental or malicious deletion. Because of this, there may be delays between when you delete something and when copies are deleted from our active and backup systems.

You can read more about Google's [data retention periods](#), including how long it takes us to delete your information.

COMPLIANCE & COOPERATION WITH REGULATORS

We regularly review this Privacy Policy and make sure that we process your information in ways that comply with it.

Data transfers

We maintain [servers around the world](#) and your information may be processed on servers located outside of the country where you live. Data protection laws vary among countries, with some providing more protection than others. Regardless of where your information is processed, we apply the same protections described in this policy. We also comply with certain [legal frameworks](#) relating to the transfer of data, such as the EU-US and Swiss-US Privacy Shield Frameworks.

When we receive formal written complaints, we respond by contacting the person who made the complaint. We work with the appropriate regulatory authorities, including local data protection authorities, to resolve any complaints regarding the transfer of your data that we cannot resolve with you directly.

California requirements

If the California Consumer Privacy Act (CCPA) applies to your information, we provide these disclosures and the [tools](#) described in this policy so you can exercise your rights to receive information about our data practices, as well as to request access to and deletion of your information. These tools allow you to review, update and delete your information, as well as export and download a copy of it. You can also [read more](#) about Google's data retention periods, and the process we follow to delete your information.

Google does not sell your personal information. We only [share your information](#) as described in this policy. Google [processes your information](#) for the purposes described in this policy, which include "business purposes" under the CCPA. These purposes include:

- **Protecting against security threats, abuse, and illegal activity.** Google uses and may disclose information to detect, prevent and respond to security incidents, and for protecting against other malicious, deceptive, fraudulent, or illegal activity. For example, to protect our services, Google may receive or disclose information about IP addresses that malicious actors have compromised.
- **Auditing and measurement.** Google uses information for analytics and measurement to understand how our services are used, as well as to fulfill obligations to our partners like publishers, advertisers, developers, or rights holders. We may disclose non-personally identifiable information publicly and with these partners, including for auditing purposes.
- **Maintaining our services.** Google uses information to ensure our services are working as intended, such as tracking outages or troubleshooting bugs and other issues that you report to us.
- **Research and development.** Google uses information to improve our services and to develop new products, features and technologies that benefit our users and the public. For example, we use publicly available information to help train Google's language models and build features like Google Translate.
- **Use of service providers.** Google shares information with service providers to perform services on our behalf, in compliance with our Privacy Policy and other appropriate confidentiality and security measures. For example, we may rely on service providers to help provide customer support.
- **Advertising.** Google processes information, including online identifiers and information about your interactions with advertisements, to provide advertising. This keeps many of our services freely available for users. You can control what information we use to show you ads by visiting your [ad settings](#).

Google also uses information to satisfy applicable laws or regulations, and discloses information in response to legal process or enforceable government requests, including to law enforcement. We provide information about the number and type of requests we receive from governments in our [Transparency Report](#).

If you have additional questions or requests related to your rights under the CCPA, [you can contact Google](#).

ABOUT THIS POLICY

When this policy applies

This Privacy Policy applies to all of the services offered by Google LLC and its [affiliates](#), including YouTube, Android, and services offered on third-party sites, such as advertising services. This Privacy Policy doesn't apply to services that have separate privacy policies that do not incorporate this Privacy Policy.

This Privacy Policy doesn't apply to:

- The information practices of other companies and organizations that advertise our services
- Services offered by other companies or individuals, including products or sites that may include Google services, be displayed to you in search results, or be linked from our services

Changes to this policy

We change this Privacy Policy from time to time. We will not reduce your rights under this Privacy Policy without your explicit consent. We always indicate the date the last changes were published and we offer access to [archived versions](#) for your review. If changes are significant, we'll provide a more prominent notice (including, for certain services, email notification of Privacy Policy changes).

RELATED PRIVACY PRACTICES

Specific Google services

The following privacy notices provide additional information about some Google services:

- [Chrome & the Chrome Operating System](#)
- [Play Books](#)
- [Payments](#)

- [Fiber](#)
- [Google Fi](#)
- [G Suite for Education](#)
- [YouTube Kids](#)
- [Google Accounts Managed with Family Link, for Children under 13 \(or applicable age in your country\)](#)
- [Voice and Audio Collection from Children's Features on the Google Assistant](#)

Other useful resources

The following links highlight useful resources for you to learn more about our practices and privacy settings.

- [Your Google Account](#) is home to many of the settings you can use to manage your account
- [Privacy Checkup](#) guides you through key privacy settings for your Google Account
- [Google's safety center](#) helps you learn more about our built-in security, privacy controls, and tools to help set digital ground rules for your family online
- [Privacy & Terms](#) provides more context regarding this Privacy Policy and our Terms of Service
- [Technologies](#) includes more information about:
 - [How Google uses cookies](#)
 - Technologies used for [Advertising](#)
 - [How Google uses pattern recognition](#) to recognize things like faces in photos
 - [How Google uses information from sites or apps that use our services](#)

ads you'll find most useful

For example, if you watch videos about baking on YouTube, you may see more ads that relate to baking as you browse the web. We also may use your IP address to determine your approximate location, so that we can serve you ads for a nearby pizza delivery service if you search for "pizza." Learn more [about Google ads](#) and [why you may see particular ads](#).

the people who matter most to you online

For example, when you type an address in the To, Cc, or Bcc field of an email you're composing, Gmail will suggest addresses based on the people you [contact most frequently](#).

phone number

If you add your phone number to your account, it can be used for different purposes across Google services, depending on your settings. For example, your phone number can be used to help you access your account if you forget your password, help people find and connect with you, and make the ads you see more relevant to you. [Learn more](#)

payment information

For example, if you add a credit card or other payment method to your Google Account, you can use it to buy things across our services, like apps in the Play Store. We may also ask for other information, like a business tax ID, to help process your payment. In some cases, we may also need to verify your identity and may ask you for information to do this.

We may also use payment information to verify that you meet age requirements, if, for example, you enter an incorrect birthday indicating you're not old enough to have a Google Account. [Learn more](#)

devices

For example, we can use information from your devices to help you decide which device you'd like to use to install an app or view a movie you buy from Google Play. We also use this information to help protect your account.

Android device with Google apps

Android devices with Google apps include devices sold by Google or one of our partners and include phones, cameras, vehicles, wearables, and televisions. These devices use Google Play Services and other pre-installed apps that include services like Gmail, Maps, your phone's camera and phone dialer, text-to-speech conversion, keyboard input, and security features.

Views and interactions with content and ads

For example, we collect information about views and interactions with ads so we can provide aggregated reports to advertisers, like telling them whether we served their ad on a page and whether the ad was likely seen by a viewer. We may also measure other interactions, such as how you move your mouse over an ad or if you interact with the page on which the ad appears.

synced with your Google Account

Your Chrome browsing history is only saved to your account if you've enabled Chrome synchronization with your Google Account. [Learn more](#)

services to make and receive calls or send and receive messages

Examples of these services include:

- Google Hangouts, for making domestic and international calls

- Google Voice, for making calls, sending text messages, and managing voicemail
- Google Fi, for a phone plan

Sensor data from your device

Your device may have sensors that can be used to better understand your location and movement. For example, an accelerometer can be used to determine your speed and a gyroscope to figure out your direction of travel.

Information about things near your device

If you use Google's Location services on Android, we can improve the performance of apps that rely on your location, like Google Maps. If you use Google's Location services, your device sends information to Google about its location, sensors (like accelerometer), and nearby cell towers and Wi-Fi access points (like MAC address and signal strength). All these things help to determine your location. You can use your device settings to enable Google Location services. [Learn more](#)

publicly accessible sources

For example, we may collect information that's publicly available online or from other public sources to help train Google's language models and build features like Google Translate.

protect against abuse

For example, information about security threats can help us notify you if we think your account has been compromised (at which point we can help you take steps to protect your account).

advertising and research services on their behalf

For example, advertisers may upload data from their loyalty-card programs so that they can better understand the performance of their ad campaigns. We only provide aggregated reports to advertisers that don't reveal information about individual people.

deliver our services

Examples of how we use your information to deliver our services include:

- We use the IP address assigned to your device to send you the data you requested, such as loading a YouTube video
- We use unique identifiers stored in cookies on your device to help us authenticate you as the person who should have access to your Google Account
- Photos and videos you upload to Google Photos are used to help you create albums, animations, and other creations that you can share. [Learn more](#)
- A flight confirmation email you receive may be used to create a "check-in" button that appears in your Gmail
- When you purchase services or physical goods from us, you may provide us information like your shipping address or delivery instructions. We use this information for things like processing, fulfilling, and delivering your order, and to provide support in connection with the product or service you purchase.

ensure our services are working as intended

For example, we continuously monitor our systems to look for problems. And if we find something wrong with a specific feature, reviewing activity information collected before the problem started allows us to fix things more quickly.

make improvements

For example, we use cookies to analyze how people interact with our services. And that analysis can help us build better products. For example, it may help us discover that it's

taking people too long to complete a certain task or that they have trouble finishing steps at all. We can then redesign that feature and improve the product for everyone.

customized search results

For example, when you're signed in to your Google Account and have the Web & App Activity control enabled, you can get more relevant search results that are based on your previous searches and activity from other Google services. You can [learn more here](#). You may also get customized search results even when you're signed out. If you don't want this level of search customization, you can [search and browse privately](#) or turn off [signed-out search personalization](#).

personalized ads

You may also see personalized ads based on information from the advertiser. If you shopped on an advertiser's website, for example, they can use that visit information to show you ads. [Learn more](#)

sensitive categories

When showing you personalized ads, we use topics that we think might be of interest to you based on your activity. For example, you may see ads for things like "Cooking and Recipes" or "Air Travel." We don't use topics or show personalized ads based on sensitive categories like race, religion, sexual orientation, or health. And we [require the same from advertisers](#) that use our services.

may link information

Google Analytics relies on first-party cookies, which means the cookies are set by the Google Analytics customer. Using our systems, data generated through Google Analytics can be linked by the Google Analytics customer and by Google to third-party cookies that are related to visits to other websites. For example, an advertiser may want to use its Google Analytics data to create more relevant ads, or to further analyze its traffic. [Learn more](#)

safety and reliability

Some examples of how we use your information to help keep our services safe and reliable include:

- Collecting and analyzing IP addresses and cookie data to protect against automated abuse. This abuse takes many forms, such as sending spam to Gmail users, stealing money from advertisers by fraudulently clicking on ads, or censoring content by launching a Distributed Denial of Service (DDoS) attack.
- The “last account activity” feature in Gmail can help you find out if and when someone accessed your email without your knowledge. This feature shows you information about recent activity in Gmail, such as the IP addresses that accessed your mail, the associated location, and the date and time of access. [Learn more](#)

detect abuse

When we detect spam, malware, illegal content, and other forms of abuse on our systems in violation of our policies, we may disable your account or take other appropriate action. In certain circumstances, we may also report the violation to appropriate authorities.

combine the information we collect

Some examples of how we combine the information we collect include:

- When you’re signed in to your Google Account and search on Google, you can see search results from the public web, along with relevant information from the content you have in other Google products, like Gmail or Google Calendar. This can include things like the status of your upcoming flights, restaurant, and hotel reservations, or your photos. [Learn more](#)
- If you have communicated with someone via Gmail and want to add them to a Google Doc or an event in Google Calendar, Google makes it easy to do so by autocompleting their email address when you start to type in their name. This feature makes it easier to share things with people you know. [Learn more](#)

- The Google app can use data that you have stored in other Google products to show you personalized content, depending on your settings. For example, if you have searches stored in your Web & App Activity, the Google app can show you news articles and other information about your interests, like sports scores, based on your activity. [Learn more](#)
- If you link your Google Account to your Google Home, you can manage your information and get things done through the Google Assistant. For example, you can add events to your Google Calendar or get your schedule for the day, ask for status updates on your upcoming flight, or send information like driving directions to your phone. [Learn more](#)

your activity on other sites and apps

This activity might come from your use of Google services, like from syncing your account with Chrome or your visits to sites and apps that partner with Google. Many websites and apps partner with Google to improve their content and services. For example, a website might use our advertising services (like AdSense) or analytics tools (like Google Analytics), or it might embed other content (such as videos from YouTube). These services may share information about your activity with Google and, depending on your [account settings](#) and the products in use (for instance, when a partner uses Google Analytics in conjunction with our advertising services), this data may be associated with your personal information.

[Learn more](#) about how Google uses data when you use our partners' sites or apps.

partner with Google

There are over 2 million non-Google websites and apps that partner with Google to show ads. [Learn more](#)

specific Google services

For example, you can delete [your blog](#) from Blogger or [a Google Site you own](#) from Google Sites. You can also delete [reviews](#) you've left on apps, games, and other content

in the Play Store.

rely on cookies to function properly

For example, we use a cookie called 'lbc's' that makes it possible for you to open many Google Docs in one browser. Blocking this cookie would prevent Google Docs from working as expected. [Learn more](#)

legal process, or enforceable governmental request

Like other technology and communications companies, Google regularly receives requests from governments and courts around the world to disclose user data. Respect for the privacy and security of data you store with Google underpins our approach to complying with these legal requests. Our legal team reviews each and every request, regardless of type, and we frequently push back when a request appears to be overly broad or doesn't follow the correct process. Learn more in our [Transparency Report](#).

show trends

When lots of people start searching for something, it can provide useful information about particular trends at that time. Google Trends samples Google web searches to estimate the popularity of searches over a certain period of time and shares those results publicly in aggregated terms. [Learn more](#)

specific partners

For example, we allow YouTube creators and advertisers to work with measurement companies to learn about the audience of their YouTube videos or ads, using cookies or similar technologies. Another example is merchants on our shopping pages, who use cookies to understand how many different people see their product listings. [Learn more](#) about these partners and how they use your information.

servers around the world

For example, we operate data centers located [around the world](#) to help keep our products continuously available for users.

third parties

For example, we process your information to report use statistics to rights holders about how their content was used in our services. We may also process your information if people search for your name and we display search results for sites containing publicly available information about you.

appropriate safeguards

For example, we may anonymize data, or encrypt data to ensure it can't be linked to other information about you. [Learn more](#)

ensure and improve

For example, we analyze how people interact with advertising to improve the performance of our ads.

Customizing our services

For example, we may display a Google Doodle on the Search homepage to celebrate an event specific to your country.

Affiliates

An affiliate is an entity that belongs to the Google group of companies, including the following companies that provide consumer services in the EU: Google Ireland Limited, Google Commerce Ltd, Google Payment Corp, and Google Dialer Inc. Learn more about the [companies providing business services in the EU](#).

Algorithm

A process or set of rules followed by a computer in performing problem-solving operations.

Application data cache

An application data cache is a data repository on a device. It can, for example, enable a web application to run without an internet connection and improve the performance of the application by enabling faster loading of content.

Browser web storage

Browser web storage enables websites to store data in a browser on a device. When used in "local storage" mode, it enables data to be stored across sessions. This makes data retrievable even after a browser has been closed and reopened. One technology that facilitates web storage is HTML 5.

Cookies and similar technologies

A cookie is a small file containing a string of characters that is sent to your computer when you visit a website. When you visit the site again, the cookie allows that site to recognize your browser. Cookies may store user preferences and other information. You can configure your browser to refuse all cookies or to indicate when a cookie is being sent. However, some website features or services may not function properly without cookies. Learn more about [how Google uses cookies](#) and how Google uses data, including cookies, [when you use our partners' sites or apps](#).

Device

A device is a computer that can be used to access Google services. For example, desktop computers, tablets, smart speakers, and smartphones are all considered

devices.

Non-personally identifiable information

This is information that is recorded about users so that it no longer reflects or references an individually-identifiable user.

IP address

Every device connected to the Internet is assigned a number known as an Internet protocol (IP) address. These numbers are usually assigned in geographic blocks. An IP address can often be used to identify the location from which a device is connecting to the Internet.

Pixel tag

A pixel tag is a type of technology placed on a website or within the body of an email for the purpose of tracking certain activity, such as views of a website or when an email is opened. Pixel tags are often used in combination with cookies.

Personal information

This is information that you provide to us which personally identifies you, such as your name, email address, or billing information, or other data that can be reasonably linked to such information by Google, such as information we associate with your Google Account.

Sensitive personal information

This is a particular category of personal information relating to topics such as confidential medical facts, racial or ethnic origins, political or religious beliefs, or sexuality.

Server logs

Like most websites, our servers automatically record the page requests made when you visit our sites. These “server logs” typically include your web request, Internet Protocol address, browser type, browser language, the date and time of your request, and one or more cookies that may uniquely identify your browser.

A typical log entry for a search for “cars” looks like this:

```
123.45.67.89 - 25/Mar/2003 10:15:32 -
http://www.google.com/search?q=cars -
Firefox 1.0.7; Windows NT 5.1 -
740674ce2123e969
```

- `123.45.67.89` is the Internet Protocol address assigned to the user by the user’s ISP. Depending on the user’s service, a different address may be assigned to the user by their service provider each time they connect to the Internet.
- `25/Mar/2003 10:15:32` is the date and time of the query.
- `http://www.google.com/search?q=cars` is the requested URL, including the search query.
- `Firefox 1.0.7; Windows NT 5.1` is the browser and operating system being used.
- `740674ce2123a969` is the unique cookie ID assigned to this particular computer the first time it visited Google. (Cookies can be deleted by users. If the user has deleted the cookie from the computer since the last time they’ve visited Google, then it will be the unique cookie ID assigned to their device the next time they visit Google from that particular device).

Unique identifiers

A unique identifier is a string of characters that can be used to uniquely identify a browser, app, or device. Different identifiers vary in how permanent they are, whether they can be reset by users, and how they can be accessed.

Unique identifiers can be used for various purposes, including security and fraud detection, syncing services such as your email inbox, remembering your preferences, and providing personalized advertising. For example, unique identifiers stored in cookies help sites display content in your browser in your preferred language. You can configure your browser to refuse all cookies or to indicate when a cookie is being sent. Learn more about [how Google uses cookies](#).

On other platforms besides browsers, unique identifiers are used to recognize a specific device or app on that device. For example, a unique identifier such as the Advertising ID is used to provide relevant advertising on Android devices, and can be [managed](#) in your device's settings. Unique identifiers may also be incorporated into a device by its manufacturer (sometimes called a universally unique ID or UUID), such as the IMEI-number of a mobile phone. For example, a device's unique identifier can be used to customize our service to your device or analyze device issues related to our services.

EXHIBIT F

Learn the basics

How Open Bidding works

Learn the life cycle of an Open Bidding request

Next: Getting started with Open Bidding

Only available in Google Ad Manager 360.

Each Open Bidding interaction between a publisher and their exchange partners is handled by Ad Manager in what is called a "server-to-server" integration. This article describes the life cycle of an ad request targeted by an Open Bidding yield group.

[The Open Bidding process](#)

[Auction dynamics](#)

[Payments](#)

The Open Bidding process

1. An ad request is triggered and information is passed to the Ad Manager ad server

Requests are sent to Ad Manager using [Google Publisher Tags](#), the [Google Mobile Ads SDK](#), or the [IMA SDK](#). Along with each request, information about the user, device and targeting is passed to Ad Manager. [Learn more](#)

Support for native inventory for yield groups is not yet available. However, if Ad Exchange wins the bid, native inventory can be served within a fixed size.

2. Ad Manager runs a unified auction to determine the best yield

The steps below are completed synchronously on the server-side as part of the [ad selection process](#).

2a. Ad Manager selects the best trafficked line item to compete in the unified auction

Using targeting and delivery pacing information, Ad Manager catalogs all eligible line items booked in the Ad Manager ad server and [selects](#) the best line item to compete via dynamic allocation in the unified auction.

2b. Ad Manager sends a bid request to targeted yield partners

Ad Manager uses yield groups to identify the list of exchanges to compete in the unified auction. Yield groups contain targeting similar to line items and include a combination of Ad Exchange, third-party

exchanges and/or Mediation ad networks. For each Open Bidding yield partner assigned to an eligible yield group, Ad Manager sends a bid request to collect their highest bids.

Requests marked as child-directed in compliance with the [Children's Online Privacy Protection Act](#) (COPPA) are not sent to Open Bidding yield partners.

[What happens if multiple yield groups are eligible for the same request?](#)

[What happens if requested ad sizes don't match the yield group targeting?](#)

[What information is sent to buyers with each Open Bidding request?](#)

[How do Ad Exchange or AdSense line items and yield groups compete?](#)

2c. Targeted yield partners run their own auction and returns their most competitive bid to Ad Manager

Yield partners utilize their own Open Bidding integration to receive the bid request from Ad Manager, run an auction according to the information provided in the request, and return their most competitive bid to Ad Manager.

2d. Ad Manager hosts a unified auction and selects a winner

Ad Manager hosts a unified [auction](#), comparing yield partner bids, the Ad Exchange bid and other direct line items via [dynamic allocation](#) to ensure that each impression maximizes yield.

3. A creative or Mediation list is returned to the publisher

After dynamic allocation and all Open Bidding auctions have completed and a winner is selected, the Ad Manager ad server returns the winning asset to the publisher for display.

- If an Ad Manager line item wins the unified auction, the Ad Manager creative is returned to the publisher.
- If an exchange bidder or an Ad Exchange buyer wins the Ad Manager unified auction, the buyer's creative is returned to the publisher.
- If a Mediation yield partner wins the Ad Manager unified auction, a Mediation list, or "chain," is returned to the publisher that includes Mediation yield partners with a CPM higher than the highest Ad Exchange or Open Bidding yield partner bid. The publisher's mobile app will then call each partner in the list in order for a creative to display. [Learn more about Mediation for mobile apps](#)

If the default CPM for a Mediation yield partner is defined as higher than the highest bid from an Open Bidding yield partner, the Mediation yield partner will have a first look to fill the impression with no guarantee they will return a bid at the default CPM price. Ensure that your default CPM values for Mediation yield partners is set realistically to optimize competition.

Auction dynamics

The real-time bids (RTB) from yield partners compete as part of [dynamic allocation](#) in a unified auction. The best Ad Manager line item rate, expected Mediation yields and exchange bids are compared at the same time and the top bid wins the auction. Ad Exchange and yield partners bid once for each impression.

All participants in the unified auction, including Ad Exchange and third-party exchanges, compete equally for each impression on a net basis. Each exchange runs its own auction independently and then submits its bid into the unified auction. Ad Manager sends the reserve price for the unified auction to all eligible Ad Exchange buyers and Open Bidding participants (including third-party exchanges or networks). The reserve price is at least the maximum of the temporary CPM calculated by Ad Manager for the best eligible guaranteed line item or the floor price configured by the publisher (as may be adjusted, at the publisher's option, by various Ad Manager optimizations). The reserve price is not set by either the value CPMs of remnant line items that are competing for the impression or any bids received from any Ad Exchange buyers or Open Bidders for the impression. No auction participant receives any information about any other party's bids prior to completion of the auction.

Example

There are three buyers. Ad Exchange Buyer 1 bids \$3.00, Ad Exchange Buyer 2 bids \$1.00, and a third-party exchange bids \$2.00 on an impression. The winner of the unified auction would be Ad Exchange Buyer 1, as \$3.00 is the highest bid submitted.

The highest net bid (which takes into account Ad Manager's revenue share) wins. The channel through which a bid is received (for example, whether through Google Ads or Display & Video 360, a third-party Authorized Buyer, Open Bidding yield partner, or a remnant line item) does not otherwise affect the determination of the winning bidder. Learn more about [unified pricing rules](#) and the [auction rules](#).

Was this helpful?

Yes

No

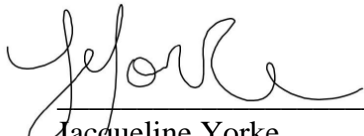
EXHIBIT G




TRANSPERFECT

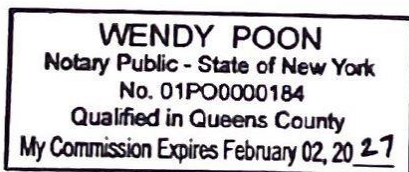
City of New York, State of New York, County of New York

I, Jacqueline Yorke, hereby certify that the document “**Aguadilla Paint Center v Esso**” is, to the best of my knowledge and belief, a true and accurate translation from Spanish into English.


Jacqueline Yorke

Sworn to before me this
January 11, 2024


Signature, Notary Public



Stamp, Notary Public

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183 D.P.R. 901, 2011 WL 6759549 (P.R.), 2011 TSPR 194

AGUADILLA PAINT CENTER, INC., CB GASOLINE SERVICE
GROUP and MICADA DE PUERTO RICO, INC., appellees,
v.
ESSO STANDARD OIL COMPANY (PUERTO RICO), petitioner.

In the Supreme Court of Puerto Rico.
Number: CC-2010-1135

Synopsis

CERTIORARI PETITION to request the review of a RESOLUTION of *Guillermo Arbona Lago, José L. Miranda de Hostos and Mildred G. Pabón Charneco*, Judges of the Court of Appeals, which concluded that after a considered analysis of the appeal submitted and the documents contained in the file, the Esso Standard Oil Company had not placed it in a position to grant the requested remedy and that the appealed opinion did not merit its intervention at this procedural stage. *The Resolution issued by the Court of Appeals is revoked and, consequently, the cause of action based on Art. 4A of the Gasoline Producers, Refiners and Distributors-Wholesalers Control Act, 23 L.P.R.A. sec. 1104a, as amended, is dismissed, before the Court of First Instance, due to lack of jurisdiction.*

Salvador Antonetti Stutts, Carlos A. Valldejuly Sastre, Carlos J. Sagardía Abreu, Pedro A. Delgado Hernández and Carla Framil Ferrén, of O'Neill & Borges, attorneys for the petitioning party; *José A. Ramos Díaz*, attorney for the appellee party; *Irene S. Soroeta Kodesh*, solicitor general, *Amir Cristina Nieves Villegas*, assistant solicitor general *Leticia Casaldue Rabell*, deputy solicitor general, and *José G. Díaz Tejera*, Assistant Secretary of the Antitrust Affairs Office of the Department of Justice, *amicii curiae*.

THE ASSOCIATE JUDGE MR. KOLTHOFF CARABALLO issued the opinion of the Court.

On this occasion, the dispute in question is relatively simple: does a private person, a gasoline retailer, have active standing to file a cause of action before the Court of First Instance against a distributor-wholesaler of that product for alleged violations of the provisions related to the duty of operational separation established in Art. 4A of the Gasoline Producers, Refiners and Distributors-Wholesalers Control Act? ¹ When addressing this question, we interpreted for the first time jointly two statutes that regulate competition in the gasoline market, namely: the Gasoline Producers, Refiners and Distributors-Wholesalers Control Act, Act No. 3 of March 21, 1978, as amended, 23 L.P.R.A. (Leyes de Puerto Rico Anotadas [*Laws of Puerto Rico Annotated*]) sec. 1101 *et seq.* (Gasoline Act), and the Antitrust *909 and Restriction of Trade Act, Act No. 77 of June 25, 1964, as amended, 10 L.P.R.A. sec. 257 *et seq.* (Antitrust Act).

¹ 28 L.P.R.A. sec. 1104a.

I

The petitioner Esso Standard Oil Company (the petitioner or Esso) is a corporation dedicated to the import, distribution and sale of gasoline in bulk, that is, a distributor-wholesaler. ² For its part, the appellees Aguadilla Paint Center, Inc., and CB Gasoline Service Group, Inc. (the appellees) are retailers, ³ that is, retail sellers of gasoline, in this case gasoline imported, distributed and sold by the petitioner Esso (brand retailers).

² See Art. 1(f) of the Gasoline Producers, Refiners and Distributors-Wholesalers Control Act (Gasoline Act), 23 L.P.R.A. sec. 1101(f).

³ See Art. 1(g) of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1101(g).

In August 2004, the appellees filed a claim before the trial court in which they charged the petitioner Esso, inter alia, with incurring in the practice of operational association, in contravention of Art. 4A of the Gasoline Act, *supra*, and Art. 7 of the Antitrust Act.⁴ In addition, they requested compensation for damages and losses, according to Art. 1802 of the Civil Code, 31 L.P.R.A. sec. 5141.

4 10 L.P.R.A. sec. 263.

After multiple procedural incidents,⁵ on March 12, 2010, Esso requested the dismissal of the claim *910 under Rule 10.2 of Civil Procedure.⁶ In its motion, Esso basically argued that the cause of action of the appellees, based on the obligation of “operational separation” established in Art. 4A of the Gasoline Act, *supra*, should be dismissed because Art. 8 of the Gasoline Act, 23 L.P.R.A. sec. 1108, establishes that a violation of Art. 4A “shall constitute an unfair or deceptive practice or act and shall be subject to the provisions of sections 257 et seq.” of the Antitrust Act, *supra*.

5 On October 1, 2004, Esso first requested the dismissal of the claim, based on the fact that Art. 4 of the Gasoline Act displaced the application of Art. 7 of the Antitrust and Restriction of Trade Act (Antitrust Act) (case DAC2005-0343). However, the Court of First Instance denied the request for dismissal, and therefore Esso appealed to the Court of Appeals (case KLCE2005-00564). The Court of Appeals denied the issuance of the order by Resolution of May 26, 2005. Esso then appealed to this Court by *certiorari*, which we denied on November 10, 2005. Such a determination became enforceable and final, thus continuing the proceedings before the Court of First Instance.

6 32 L.P.R.A. Ap. III.

The trial court declared Esso’s application for dismissal “inadmissible” by concluding “that a private entity such as the plaintiff may file an action under Article 7 of the [Gasoline Act, 23 L.P.R.A. Sec. 1107] on competition in the gasoline market, before the Court of First Instance.”⁷ Dissatisfied, Esso appealed to the Court of Appeals through a *certiorari appeal*. This venue concluded that, after a considered analysis of the appeal submitted and the documents contained in the file, Esso had not placed it in a position to grant the requested remedy and that the appealed opinion did not merit its intervention in that procedural stage.

7 Appendix to the Petition for *certiorari*, p. 179.

In view of this ruling, Esso appeared before this venue by means of an appeal of *certiorari* outlining the following points of error:

The Court of Appeals erred by not issuing the requested order and not recognizing that Art. 12(a) of the Antitrust Act excludes private claims for alleged operational association under Art. 4A of the Gasoline Act.

The Court of Appeals erred by not issuing the requested order and not ordering the dismissal of the claim under Art. 4A of the Gasoline Act, since the plaintiffs do not have active standing to initiate it and, therefore, the Court of First Instance lacks jurisdiction over the matter to address it. *911

The Court of Appeals erred in determining that Esso did not place it in “conditions of being able to grant the requested remedy,” since the criteria for the issuance of the order contained in Rule 40 of the Rules of the Court of Appeals are met. Request for *certiorari*, p. 8.

Once the *certiorari* was issued, we stopped the proceedings and ordered the parties to submit their corresponding arguments.⁸ With the benefit of the position of all parties, we now proceed to rule.

8 We also ordered the Solicitor General of Puerto Rico to appear and express her position on the dispute, as *amicus curiae*; the Solicitor General did so.

II

A. Gasoline Producers, Refiners and Distributors-Wholesalers Control Act

[1] As it arises from its Statement of Reasons, the approval of the Gasoline Producers, Refiners and Distributors-Wholesalers

Control Act, Act No. 3 of March 21, 1978, as amended, 23 L.P.R.A. sec. 1101 *et seq.*, had the intention, among others, to combat the problem “that arises from the fact that the control of energy product supplies and, especially, of gasoline and other petroleum and natural gas fuels, continue to be concentrated in hands of a few who dominate all aspects of production, refining and marketing of these products.”⁹ In response to this the legislator sought to eliminate the competitive advantages existing in the gasoline or special engine fuels market, as well as to have oversight of that process and to establish measures for the fulfillment of the purposes pursued by said statute. In *912 accordance with this purpose, the legislator classified infractions and set penalties for the violators of the statute.

9 See Statement of Reasons for the Gasoline Act, 1978 Puerto Rico Laws 16-17.

Thus, the Legislative Assembly determined that, in the absence of competitive uniformity, it was necessary to institute careful safeguards that prevented the actions of oil companies and distributors of energy products aimed at monopolizing the points of public distribution of gasoline.¹⁰ In addition, the Gasoline Act seeks to protect the retail fuel sales industry from any discriminatory practices that seek to eliminate the individual gasoline retailer from market competition, as well as from those discriminatory practices and control over the price structure aimed at favoring certain retailers to the detriment of others and the consumer.¹¹

10 Statement of Reasons for the Gasoline Act, *supra*.

11 *Id.*

[2] On the other hand, just a few months after the passage of the Gasoline Act, the Legislature approved, as an *addendum* to the latter, Act No. 73 of June 23, 1978, as amended, 23 L.P.R.A. sec. 1131 *et seq.* (Act No. 73), addressing the “Regulation and Control of the Gasoline Industry.” With the passage of Act No. 73, “the gasoline industry in all its facets was declared as being in the public interest.”¹² In doing so, it was recognized that the gasoline industry is a fundamental element for the security and well-being of the People of Puerto Rico. This is because the availability of that product was at that time—and obviously continues to be so today—critical for the normal functioning of our economy and the continued development of the country’s daily activities.

¹³ *913

12 23 L.P.R.A. sec. 1131.

13 See Statement of Reasons for the Gasoline Industry Regulation and Control Act, Act No. 73 of June 23, 1978, as amended, 23 L.P.R.A. sec. 1131 *et seq.* (Act No. 73).

In harmony with the Gasoline Act, the purpose of Act No. 73 is to effectively regulate certain aspects of the gasoline industry and thus ensure that the interests of the People of Puerto Rico are adequately protected from harmful activities that take place at any of the different operational levels of that industry.¹⁴ In so acting, the legislator indicated that the Government is responsible for ensuring that there is a stable situation within all activities that affect the general well-being of the public, as well as to ensure the good order of the community and the protection of the best interests of the economic and government system.¹⁵

14 *Id.*

15 *Id.*

To fulfill its purposes, Act No. 73 established certain obligations and prohibitions detailed in its Art. 3.¹⁶ Among them, and as an example, it was prohibited that in the relationships between distributors-wholesalers and retailers, that there be simultaneously lease and lease-back agreements, when said relationship has the effect or purpose of canceling the obligation of payment between the parties or of unreasonably restricting the right to free contracting or to the disposal of the property.¹⁷ In addition, Act No. 73 established the obligation for any distributor-wholesaler to present at the Antitrust Affairs Office (O.A.M., Oficina de Asuntos

Monopolísticos) ¹⁸ a legible copy of any contract establishing a commercial relationship with a retailer, within ten days after its execution, as well as any subsequent amendment or modification thereof. *914

16 23 L.P.R.A. sec. 1131.

17 23 L.P.R.A. sec. 1133(a).

18 As we will detail below, the Antitrust Affairs Office (O.A.M.) is an entity attached to the Department of Justice, with powers for the administration of legislation on monopolistic practices, as well as for the oversight thereof.

Collectively, and to ensure the fulfillment of its objectives and purposes, Act No. 73 empowered various government agencies and imposed on them the responsibility to perform various functions. These agencies include the Department of Commerce, ¹⁹ the Public Service Commission, the Planning Board, the Regulations and Permit Administration, and *the Department of Consumer Affairs* (D.A.Co., *Departamento de Asuntos del Consumidor*). ²⁰

19 Currently, Department of Economic Development and Commerce.

20 23 L.P.R.A. sec. 1133(c)–(h).

Finally, it is necessary to consider what Art. 5 of Act No. 73 establishes in relation to the Antitrust Act, *supra*:

Sec. 1134. Enforcement of the Antitrust Act

Nothing herein provided or promulgated by regulation or otherwise, pursuant to the provisions of Sections 1131 to 1135 of this title, shall be construed as excepting or conferring immunity as to the applicability of the provisions of Sections 257 to 274 of Title 10. ²¹

21 23 L.P.R.A. sec. 1134.

[3] In summary, it is evident that from its inception, both the Gasoline Act and its *addendum*, Act No. 73, provided not only for the protection of the retail fuel sales industry of any discriminatory practice to the detriment of the best interests of the people of Puerto Rico, but they expressly recognized the impact of the Antitrust Act in such a process. Thus, both laws provided for the actions of the Puerto Rico Department of Justice, through its O.A.M., to be cardinal in compliance with its oversight provisions. In addition, the legislator arranged for other government agencies, such as the D.A.Co, to also participate.

[4] In keeping with all of the foregoing, Art. 2(a) of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1102(a), establishes that *915

... no producer, refiner or distributor-wholesaler will acquire or establish, open, operate or recover to operate any gasoline retail service station to be operated with personnel of their own company or subsidiary company, agent, commission agent, or under contract with any individual or legal entity, operating or administering said retail service station by means of a paid agreement or arrangement with said producer, refiner or distributor-wholesaler. The gas sales service station may only be operated by a retailer.

[5] Likewise, Art. 4 of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1104, establishes that

any oil producer or refiner or distributor-wholesaler of petroleum products that supplies gasoline and/or special fuels to service stations for the retail sale of such products shall be obliged to uniformly provide to all retailers of the sale of gasoline and/or special fuels to whom it supplies, any discount, deduction, decrease or reduction in prices granted directly or indirectly. ²²

22 In turn, Art. 5 of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1105, provides that any oil producer or refiner or distributor-wholesaler of petroleum products that supplies gasoline or special fuels to retailers, must uniformly apply to all retail sellers to whom it supplies, the rental of equipment and signage, when they are provided directly or indirectly to said retailers by that producer, refiner or distributor-wholesaler. Meanwhile, Art. 6 of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1106, orders the uniform distribution of fuel in periods of shortage or decrease in the supply or availability of gasoline or special fuels. Art. 7 of the aforementioned law, 23 L.P.R.A. sec. 1107, prohibits a retail seller of fuel, knowingly, from requesting or inducing the concession, or reduction in prices or in the rental of equipment and signage, in violation of Arts. 4 and 5 of the Gasoline Act, 23 L.P.R.A. sec. 1107.

[6] Having stated the above, directly related to the dispute at hand, we analyze Art. 4A of the Gasoline Act, *supra*, which provides the following:

Sec. 1104a . Operational Separation

No refiner, oil producer or distributor-wholesaler may, by agreement, arrangement, contract, corporate operational scheme, with any retailer and/or individual or legal entity, or in any other way directly operate a *916 gasoline retail service station in a way that prevents its complete operational separation. No refiner, oil producer or distributor-wholesaler may, by agreement, arrangement, contract, operating [sic] corporate scheme, with any retailer and/or individual or legal entity, or in any other way directly impose, require, fix or limit the profit margin and/or retail price of gasoline and/or special fuels at the retail service station.

[7] Art. 4A of the Gasoline Act, *supra*, is the result of an amendment made to this statute by Act No. 157-1996 (Act No. 157) and another amendment to the article itself - which adds the second sentence - by the approval of Act No. 74-2005. The article establishes the “operational separation” or, what is the same, prohibits the so-called “operational association” of gasoline wholesalers with stations for retail sale. Act No. 157 also amended the Gasoline Act to establish that “operational separation” is the cessation by a refiner, oil producer, or distributor-wholesaler “of directly participating in the operation or commercial exchange activities of the retail sale of gasoline and special fuels at retail service stations.”²³

23 23 L.P.R.A. sec. 1101(j).

[8] On the other hand, and cardinal to the dispute at hand, we must also consider Art. 8 of the Gasoline Act, which provides the following:

Sec. 1108. Violation of fair competition

Any violation of Sections 1102, 1102a, 1104, 1104a, 1105 and 1105a of this title *shall constitute an unfair [or] deceptive act or practice* and shall be subject to the provisions of *Sections 257 et seq. of Title 10*. (Emphasis added.)²⁴

24 23 L.P.R.A. sec. 1108.

[9] Art. 8 of the Gasoline Act, *supra*, is intended for two intimately linked purposes. First, to institute *917 as an “unfair or deceptive practice or act” any violation of Arts. 2, 2A, 4, 4A, 5 and 5A of the Gasoline Act. Second, to *establish that any violation of the aforementioned articles will be subject to the provisions of the Antitrust Act*. Complementing this provision, Art. 10 of the Gasoline Act provides that “[t]he fulfillment of the purposes and provisions of sections 1101 to 1110 of [the Gasoline Act] shall be the responsibility of the Secretary of Justice, through the Antitrust Affairs Office of said Department.”²⁵

25 23 L.P.R.A. sec. 1110.

In its original version, Art. 8 of the Gasoline Act, *supra*, provided as follows:

Any violation of Arts. 4, 5, 6 and 7 shall constitute an unfair or deceptive act or practice and shall be subject to the provisions of Article 3 of Act No. 77 of June 25, 1964.²⁶

²⁶ Art. 8 of Act No. 3 of March 25, 1978.

Note that this article originally referred to violations of Art. 4 of the Gasoline Act, *supra*, which establishes the obligation of any oil producer or refiner or distributor-wholesaler of petroleum products to provide uniformity for any discount, deduction, decrease or reduction in retail prices; violations of Art. 5, which establishes the obligation of any oil producer or refiner or distributor-wholesaler of petroleum products to provide uniformity in the rental of equipment and signage to retailers, and violations of Art. 6, which establishes the obligation of any oil producer or refiner or distributor-wholesaler of petroleum products to institute indiscriminate proration and uniform distribution among all retailers to whom gasoline is supplied for sale, during times of shortages, or decrease in the supply or availability of gasoline or special fuels. On the other hand, Art. 8 in its original version also referred to violations of Art. 7, *918 which establishes a prohibition on the retailer to request or induce the concession or reduction in prices in the rental of equipment and signage. As can be seen, each provision to which Art. 8 in its original version referred—Arts. 4 to 7—contained a *prohibition* whose violation constituted “an unfair or deceptive practice or act.”

However, since its original wording, Art. 8 of the Gasoline Act, *supra*, has undergone two amendments. The first of them through the approval of Act No. 157-1996 (Act No. 157), which in its Statement of Reasons indicated the following (referring to Act No. 3 of 1978, Gasoline Act):

In 1978, the Legislative Assembly passed legislation *aimed at protecting gasoline retailers from potential predatory practices by wholesale gasoline distribution companies*. One of those laws, Act No. 3 ...provided that *no wholesale company could directly operate gasoline stations*, subject to certain exceptions. *The responsibility for monitoring compliance with this law was assigned to the Secretary of Justice through its Antitrust Affairs Office.* (Emphasis added)²⁷

²⁷ See Statement of Reasons of Act No. 157-1996 (Act No. 157), 1996 Puerto Rico Laws 683.

As can be seen, in the Statement of Reasons of Act No. 157, the Legislative Assembly mentioned the Gasoline Act among those laws whose intention is to “protect gasoline retailers from possible *predatory practices by wholesale gasoline distribution companies*.” (Emphasis added.)²⁸ In addition, it noted that the Gasoline Act “provided that no wholesale company could directly operate gasoline stations, subject to certain exceptions.”²⁹ In such a way that Act No. 157, in its Statement of Reasons, recognized as a predatory practice, that is, harmful to retailers and prohibited by the Gasoline Act, that in which the *919 wholesaler directly operated a gasoline station. It also pointed out that the oversight of that prohibition was a matter that the *Antitrust Affairs Office* would be responsible for.

²⁸ Id.

²⁹ Id.

On the other hand, in its last paragraph, the Statement of Reasons of Act No. 157 also indicates as its purpose to amend the Gasoline Act so that the complete separation between retail sales operations and those of wholesalers and gasoline refiners is established. The aforementioned text provides as follows:

This measure aims to close the exception gaps that, because they were opened in 1978, have limited the achievement of public objectives in this industry. *Specifically, the complete separation between retail sales operations and those of wholesalers and gasoline refiners is established, with a deadline for the necessary separations to be carried out until it is achieved.* (Emphasis added.)³⁰

30 Id.

As part of the amendments made to the Gasoline Act aimed at establishing that “complete separation between operations” of retailers and wholesalers, Act No. 157 created the aforementioned Art. 4A, which prohibits any act that prevents the so-called “operational separation.” The definition of “operational separation,” which Act No. 157 itself also established as one of the amendments to the Gasoline Act, *supra*, states that it “occurs when a... wholesale distributor ceases to participate directly in the operation or commercial exchange activities of the retail sale of gasoline... at retail service stations.”³¹

31 23 L.P.R.A. sec. 1101(j).

However, the amendment to Art. 8 of the Gasoline Act, *supra*, produced by Act No. 157, changed the majority of the articles which referred to the version of the original article. The version of Art. 8 after the amendment of Act No. 157 became as follows:
*920

Any violation of Articles 1, 2, 3, 4 and 5 shall constitute an unfair or deceptive act or practice and shall be subject to the provisions of Article 3 of Act No. 77 of June 25, 1964, as amended... 1996 Puerto Rico Laws 686.

In analyzing the above text we have to conclude that the aforementioned amendment to Art. 8 that was introduced by Act No. 157 produced illogical changes in some cases and inconsequential changes in others. By this we mean the following. Firstly, it makes no sense that a law whose Statement of Reasons indicates that its purpose is to amend the Gasoline Act to establish a complete separation between retail and wholesale sales operations, thus avoiding “possible predatory [monopolistic] practices by wholesale gasoline distribution companies”; which creates Art. 4A of “operational separation” for those purposes, and which acknowledges that the oversight for such compliance rests on the *expertise* of the O.A.M., should not include Art. 4A as part of the amendment to the aforementioned Art. 8 of the Gasoline Act, *supra*.

On the other hand, the amendment to Art. 8 of the Gasoline Act, *supra*, which introduced Act No. 157 also included, as “unfair or deceptive practice or act,” any violation of Art. 1 of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1101. However, Art. 1 of the Gasoline Act, *supra*, only established at that time—and currently establishes—a series of definitions of terms, none of which constitutes a prohibition *per se*. Therefore, no one could commit a violation of Art. 1 of the Gasoline Act, *supra*, since such an article did not prohibit and does not prohibit anything.

Finally, the same thing happened with Art. 3 of the Gasoline Act, *supra*, 23 L.P.R.A. sec. 1103. Such an article established at that time—and this remains the same currently—the creation of the “Inter-agency Committee on the Gasoline Industry.” As was the case with Art. 1 (“Definitions”), Art. 3 did not constitute any prohibition, so the inclusion *921 of that article in the enumeration established by the amendment of Act No. 157 to Art. 8 of the Gasoline Act, *supra*, was totally inconsequential.

As such, the question that is applicable is what the final wording and approval of Art. 8 of the Gasoline Act, *supra*, was due to, after the amendment made by Act No. 157. Although the legislative history does not give us much to go on in this regard, we do

find that the first version of amendment to this article (which was presented in the Bill of the House of Representatives) indicated that “any violation of Articles 2, 4, 4A, 5, 5A, 6, 6A, and 7, would constitute an unfair or deceptive practice or act.” (Emphasis in the original and added.)³²

32 See Bill 2288 of January 30, 1996, 7th Ordinary Session, 12 Legislative Assembly, p. 7.

On the other hand, when codifying the version of the second and last amendments made to this article through Act No. 74-2005, the following codifying note is indicated in its history:

The amending law of 1996 appears to have referred to Arts. 1, 2, 3, 4 and 5 of the law itself instead of the articles amended or added thereto, namely: Arts. 1, 2, 4A, 5A and 8, i.e., *sec. 1101, 1102, 1104a, 1105a and 1108 of this title. However, because it is similar to the content of the sections originally referred to in the 1978 law, it could be the case that the 1996 law attempted to refer to Arts. 4, 4A, 5 and 5A of the law, sec. 1104, 1104a, 1105 and 1105a of this title.* (Emphasis added.) 23 L.P.R.A. sec. 1108 n.

In considering all of the above, we are sure that the version of Art. 8 of the Gasoline Act, *supra*, which arose as a result of the amendment made by Act No. 157 was the product of a likely typographical error, which did not reflect the intention or will of the legislator. As the codification note of the history of this article suggests, we understand that the intent of Act No. 157 was to include Arts. 4, 4A, 5 and 5A in version *922 of Art. 8 of the Gasoline Act, *supra*, and its exclusion responds to a mere administrative error (*clerical error*).

[10] We determined the foregoing, considering the legal hermeneutical standard that we reiterated in *Passalacqua v. Mun. of San Juan*, 116 D.P.R. 618, 623 (1985), where we indicated the following:

In *Roig Commercial Bank v. Buscaglia, Tes.*, 74 D.P.R. 986, 998 (1953), this Court determined that if a word, phrase or provision has been approved by mistake or error, *especially if it is contrary to the rest of the law or would limit its effectiveness, it can be eliminated. We understand that under the same circumstances a phrase or words can be added, so that legislative intent can be fulfilled.* (Emphasis added.)

So, being consistent with what our guideline has been in terms of legal interpretation when confronting situations in a law such as the case in question, we determined that the legislator intended that Art. 4A was to be part of those listed in Art. 8 of the Gasoline Act, *supra*, since the approval of Act No. 157; this must be interpreted in this way.

[11] To finalize the analysis of the articles of the Gasoline Act that are necessary to consider for the resolution of the case in question, we must refer to Art. 7 of the aforementioned law, 23 L.P.R.A. sec. 1107. This article provides the following:

Sec. 1107. Obligation of the retailer

No individual or legal entity *operating a retail service station* may knowingly request or induce the concession, or reduction in prices or the rental of equipment and signage, in violation of sections 1104 and 1105 of this title. (Emphasis added.)

As it clearly arises from its text—including from the title—this article is intended *only* for the retail seller of fuel, prohibiting it from knowingly inducing or requesting a *923 producer or distributor-wholesaler (among others) to grant or reduce prices or the rental of equipment and signage, in violation of Arts. 4 and 5 of the Gasoline Act, *supra*. In other words, the object of prohibition is that *retailer* who intends to request or induce a producer or distributor to provide different prices on gasoline or special fuels,

other than those provided to other retailers, in violation of Art. 4 of the Gasoline Act, *supra*. Likewise, the article is intended only for the *retailer* who requests or induces the producer or distributor-wholesaler (among others) to provide it with a different rental cost for equipment and signage than that provided to other retailers, in violation of Art. 5 of the Gasoline Act, *supra*.

B. Antitrust Act

[12] Antitrust laws, as well as those that seek to establish fair competition and that establish restrictions on trade, constitute statutes that require extreme care both in their legislation and in their legal interpretation. The objective that inspires this type of statute is based on pressing interests and, as such, indispensable for the economy of the jurisdiction in which they are applied.

Thus, in *Colón Cabrera v. Caribbean Petroleum*,³³ we expressed in relation to our Antitrust Act that, “considering that it is a matter that affects the foundation of our democratic society, we understand that the purposes underlying said legislation are of the highest hierarchy and constitute an urgent interest of the State.”

33 170 D.P.R. 582, 595 (2007).

The main purpose of this type of statute lies in the fundamental economic principle that provides for the preservation of freedom of competition and the hindrance of *924 any practice that harms the development of the different markets. Therefore, such statutes aim to eradicate abusive, unfair and monopolistic acts that tend to limit commercial activity, thus favoring free and open competition in the markets.³⁴

34 J. von Kalinowski, *Antitrust Laws and Trade Regulation*, Sec. 1.02, Second ed. (2004).

[13] In Puerto Rico, the Antitrust Act was drafted based on two federal statutes: the Sherman³⁵ Act and the Clayton Act.³⁶ In fact, this is also the pattern in most states of the Union that also have statutes that in some manner regulate competition, monopolies, and trade restrictions, and that contain provisions comparable to those of federal antitrust laws.

35 15 U.S.C.A. seq. 1 *et seq.*

36 15 U.S.C.A. seq. 12 *et seq.*; *P.R. Fuels, Inc. v. Empire Gas Co., Inc.*, 149 D.P.R. 691, 706–707 (1999).

Our Antitrust Act was enacted “to ensure for the Public in general, and small merchants in particular, the benefits of free competition.”³⁷ As we pointed out in *Colón Cabrera v. Caribbean Petroleum*, *supra*, p. 594, the purpose of the Antitrust Act is to

37 *G.G. & Supp. Corp. v. S. & F. Sys., Inc.*, 153 D.P.R. 861, 869 (2001).

... avoid “collusion between businesses to dominate the market, [the] hoarding of raw materials, improper increases in prices resulting from a monopolistic position, discriminatory practices in customer relations [and the] extreme concentration of economic activity and wealth in some large consortia of companies.” (Square brackets in original.)

However, with regard to the dispute in question, it is necessary to consider, in particular, Articles 3(a) and 12(a) of the Antitrust Act. *925

C. Art. 3(a) of the Antitrust Act

[14] Where relevant, Art. 3(a) of the Antitrust Act³⁸ provides that “unfair methods of competition, as well as *unfair or deceptive practices or acts* in business or trade” shall be illegal. (Emphasis added.) Likewise, subsection (c) of this article establishes the following:

38 10 L.P.R.A. sec. 259(a).

(c) Without prejudice to the power to use the means authorized by sec. 269 of this title [*injunctions*], the Antitrust Affairs Office may file [sic] and process administrative complaints with the Department of Consumer Affairs to avoid, prevent and stop violations of subsection (a) of this section or the regulations approved in accordance with subsection (b) thereof.³⁹

39 10 L.P.R.A. sec. 259(c).

Unlike other provisions of the Antitrust Act, Art. 3 was based on Sec. 5(a)(1) of the law created by the Federal Trade Commission (15 U.S.C.A. sec. 45), which, like Art. 3(a), declares unfair methods of competition and unfair or deceptive acts or practices affecting trade to be illegal.⁴⁰

40 Session Diary of the Senate of Puerto Rico of May 22, 1964, page 1706.

[15–16] On the other hand, subsection (c) of Art. 3 of the Antitrust Act, 10 L.P.R.A. sec. 259(c) expressly empowers the Antitrust Affairs Office of the Department of Justice to act as an oversight body before the D.A.Co. regarding any violation by “unfair methods of competition, as well as unfair or deceptive practices or acts in business or trade.” The Antitrust Affairs Office was created through Art. 16 of the aforementioned statute to implement the purpose that inspires it.⁴¹ The O.A.M. is an entity attached to the Department *926 of Justice, with powers for the administration of legislation on monopolistic practices, as well as for the oversight thereof. In addition, it has the power to enact regulations and proscribe actions that constitute unfair methods of competition and unfair or deceptive practices or acts in business or trade.⁴² In short, the O.A.M. is an entity created with a delegation of broad powers that allows the State to have the precise investigative instruments to fulfill the oversight purpose of the Antitrust Act.⁴³ There is no doubt that, according to Art. 3 of the Antitrust Act, *supra*, the O.A.M. is the government body with the ministerial duty to supervise and ensure compliance with that law.

41 Art. 16(a) of the Antitrust Act, 23 L.P.R.A. sec. 272(a).

42 10 L.P.R.A. sec. 259(b).

43 *Colón Cabrera v. Caribbean Petroleum*, *supra*, p. 594.

[17] However, note that Art. 3 of the Antitrust Act, *supra*, does not confer adjudicative powers on the O.A.M. That is, the O.A.M. is not made responsible for adjudicating the dispute, but rather for investigating and presenting the cause of action. Subsection (c) of Art. 3 of the Antitrust Act, *supra*, provides not only that it is before D.A.Co that the O.A.M. may file and process complaints seeking to avoid, prevent and stop violations of subsection (a) of the article itself, but that the rest of the article establishes an entire procedural and appeal scheme that must be followed before that agency and of which we will mention some details.

Thus, subsection (c) of Art. 3 of the Antitrust Act, *supra*, establishes that once the defendant has been notified of the complaint against it, the D.A.Co. must hold a hearing and resolve the dispute between the parties, granting the most appropriate remedy as soon as possible.⁴⁴ Meanwhile, subsection (d) establishes that the O.A.M. will have a term of thirty days to appeal for review *927 the decision of the D.A.Co. to the Court of First Instance, if the result goes against it.⁴⁵ For its part, subsection (e) establishes how such an appeal for review will be formalized before the Court of First Instance, of which the defendant and the D.A.Co. must be notified, which will have a term of fifteen days to request an intervention.⁴⁶

44 23 L.P.R.A. sec. 259(c).

45 23 L.P.R.A. sec. 259(d).

46 23 L.P.R.A. sec. 259(e).

Subsection (g) states that the Court of First Instance will review the decision of the D.A.Co. based on the administrative record that was submitted to the agency and only with respect to the conclusions of law. *In addition, the D.A.Co.’s factual determinations “will be conclusive to the court if supported by substantial evidence.”*⁴⁷

47 23 L.P.R.A. sec. 259(g).

It is evident that by the direct reference to that agency, by the very detailed procedure established before it, and considering the nature of the causes of action that the antitrust disputes involve —especially in the face of allegations of unfair or deceptive practices or acts— the legislator decided that it was the D.A.Co. that had the exclusive primary jurisdiction to process and

adjudicate any complaint filed by the O.A.M.

[18] The exclusive primary jurisdiction of the D.A.Co. in this matter is further confirmed by the Internal Law of the Department of Consumer Affairs itself, Act No. 5 of April 23, 1973, as amended,⁴⁸ which in its Art. 6(x) includes among the powers and faculties that the Secretary of Consumer Affairs will exercise the “adjudication of the complaints that the Antitrust Affairs Office of the Department of Justice files [sic] and processes under the provisions of sec. 259 [Art. 3] of *928 Title 10.”⁴⁹ As we see, in harmony with Art. 3(c) of the Antitrust Act, subsection (x) of Art. 6 of the Internal Law of the D.A.Co. confers jurisdiction on that agency to adjudicate the complaints presented to it by the O.A.M. under Art. 3 of the Antitrust Act.

⁴⁸ 3 L.P.R.A. sec. 341e(x).

⁴⁹ Id.

[19] In conclusion, it is the Department of Justice, through the O.A.M., to which, according to the legislative mandate, is responsible for directly overseeing the violations of the Antitrust Act, specifically with regard to the violations of the provisions of unfair trade practices, including any unfair or deceptive practice or act. In addition, we conclude that, in accordance with Art. 3 of the Antitrust Act, *supra*, the D.A.Co. has the exclusive primary jurisdiction in any antitrust cause of action undertaken by the O.A.M., whose cause of action is based on any unfair or deceptive practice or act.

D. Art. 12(a) of the Antitrust Act

[20–21] As we have already established, a violation of the prohibition of operational association established in Art. 4A of the Gasoline Act, *supra*, constitutes an “unfair or deceptive practice or act,” an action that would be subject to the provisions of our Antitrust Act, in accordance with Art. 8 of the Gasoline Act, *supra*. Therefore, in view of the allegation of a violation of the provision that requires such “operational separation,” refer to those articles of the Antitrust Act that concern “unfair or deceptive practices or acts” in trade, to resolve any questions on this matter. We have already analyzed Art. 3 of the Antitrust Act, *supra*, so it now remains to discuss Art. 12 of the aforementioned statute, specifically subsection (a), which provides: *929

Sec. 268. Suits by injured persons

(a) Anyone who is injured in their business or property by another person, due to acts, or attempted acts, prohibited or declared illegal by the provisions of this chapter, *except for those of seq. 259* and 261 of this title, may sue for such acts before the Court of First Instance and shall be entitled to recover three (3) times the amount of the damages and losses caused, plus the costs of the proceeding and a reasonable amount for attorney’s fees. (Emphasis added.)⁵⁰

⁵⁰ 10 L.P.R.A. sec. 268(a).

Art. 12(a) of the Antitrust Act, *supra*, establishes the so-called “treble damages remedy” for any person who suffers damages as a result of antitrust acts. The legal action to recover such damages must be initiated within four years, counted from the origin of the cause of action.⁵¹ However, it is *clearly deduced from the text of this subsection (a) of Art. 12 of the Antitrust Act, supra, that, without any distinction, it exempts from its jurisdictional scope any cause of action that is intended or has to be initiated under sec. 259 of the Antitrust Act itself; that is, any cause of action for unfair or deceptive practices or acts.*

⁵¹ 10 L.P.R.A. sec. 268(c).

III

In the present case, we do not have any doubts regarding the fact that a private person, whether an individual or legal entity, lacks active standing to file a cause of action against a distributor-wholesaler for alleged violation of Art. 4A of the Gasoline Act, *supra*. This clearly arises from each statute concerned. As follows.

Art. 8 of the Gasoline Act, *supra*, establishes as a “unfair [or] deceptive practice or act” any violation of Art. 4A of the

aforementioned law and states that such violation *930 "will be subject to the provisions of [sec. 257 et seq.](#) of Title 10," that is, to section 257 "and the following" of the Antitrust Act.

For its part, Art. 3(a) of the Antitrust Act ([10 L.P.R.A. sec. 259\(a\)](#)) declares as illegal "unfair or deceptive practices or acts," which is precisely the conduct described and referred to the scope of that law by Art. 8 of the Gasoline Act, *supra*. Note, then, that there is clear identification or concordance in the language used in both provisions. So, by the clear language of both statutes—Gasoline Act and Antitrust Act—it is evident that the legislator provided, through Art. 8 of the Gasoline Act, *supra*, that any violation of Art. 4A of that law was to be included, covered or under the jurisdictional scope of Art. 3 of the Antitrust Act, *supra*.

[22] Once we determine that the conduct prohibited by Art. 4A of the Gasoline Act, *supra*, is under the jurisdictional scope of Art. 3 of the Antitrust Act (10 L.P.R.A. sec. 259), we face Art. 12(a) of the Antitrust Act, *supra*, which, in turn, expressly exempts from its jurisdictional scope any action that is filed under the aforementioned section 259. Therefore, any private person who suffers damages due to actions prohibited by the Antitrust Act has a treble damages remedy under Art. 12(a) of the Antitrust Act, *supra*, *except when their cause of action is based on Art. 3 of the Antitrust Act (10 L.P.R.A. sec. 259)*.⁵² This being the case, obviously, *any cause of action under Art. 4A of the Gasoline Act, supra, is excluded from the possibility of the remedy *931 of treble damage provided by Art. 12(a) of the Antitrust Act, supra.*

52 Art. 12(a) also includes within that exception the provisions of [10 L.P.R.A. sec. 261](#).

[23] In this context, Esso argues that the court of instance erred by not dismissing, under Rule 10.2 of Civil Procedure, the claim of the appellees based on Art. 4A of the Gasoline Act.⁵³ And it is correct. Rule 10.2 of Civil Procedure of 1979 (32 L.P.R.A. Ap. V) establishes privileged defenses that, as such, can be presented at any time in the proceeding.⁵⁴ That is, such defenses are not lost if they are not put forth in the response to the claim or earlier. Among those privileged defenses, Rule 10.2 includes the lack of jurisdiction over the subject matter.⁵⁵

53 See pleading of the petitioning party, p. 15.

54 R. Hernández Colón, *Legal Practice of Puerto Rico: civil procedural law*, 4th ed., San Juan, Ed. Lexisnexis, 2007, Sec. 2601, p. 234.

55 The new Rule 10.8(c) of Civil Procedure, 32 L.P.R.A. Ap. V, points out that "whenever it arises, at the direction of the parties or otherwise, that the court lacks jurisdiction over the matter, it will dismiss the lawsuit."

[24] Recently, in *Gonzalez v. Mayagüez Resort & Casino*, 176 D.P.R. 848, 855 (2009), we reiterated our guideline in the sense that the absence of jurisdiction on the matter:

(1) is not capable of being remedied; (2) the parties may not voluntarily confer it on a court of law nor may the court give it to them; (3) entails the nullity of the opinions issued; (4) imposes on the courts the unavoidable duty to sound out their own jurisdiction; (5) imposes on the appellate courts the duty to examine the jurisdiction of the venue from which the appeal comes, and (6) may occur at any stage of the procedure, at the request of the parties or by the court *ex officio*.

[25] On the other hand, a plaintiff party has active standing if it meets the following requirements: "(1) it has suffered clear and palpable damage; (2) the aforementioned damage is real, immediate and precise, and not abstract or hypothetical; (3) *932 there is a connection between the damage caused and the cause of action exercised, and (4) *the cause of action arises under the protection of the Constitution or of a law.*" (Emphasis added.)⁵⁶ Therefore, considering that, as we have established, a private person cannot file a cause of action for alleged violation of Art. 4A of the Gasoline Act, *supra*, the appellees lack a cause of

action that arises under the protection of a law. Thus, the court of instance certainly lacked jurisdiction over the matter to address such cause of action and should have dismissed it, in accordance with Rule 10.2 of Civil Procedure of 1979, *supra*.

56 *Col. Peritos Elec. v. A.E.E.*, 150 D.P.R. 327, 331 (2000). See also *Hernandez Torres v. Governor*, 129 D.P.R. 824, 835 (1992).

[26] On the other hand, we recently established in *SLG Semidey Vázquez v. ASIFAL*, 177 D.P.R. 657, 676 (2009), that “if [a] statute confers jurisdiction on the administrative body, it is an (exclusive) statutory jurisdiction.”⁵⁷ In that case, we also established the following:

57 See also Mr. Fernandez, *Derecho administrativo y Ley de Procedimiento Administrativo Uniforme [Administrative Law and Uniform Administrative Procedure Law]*, 2nd ed. rev., Bogotá, Ed. Forum, 2001, Sec. 8.3, p. 437 esc. 1.

The concept of *statutory or exclusive jurisdiction* relates to concurrent primary jurisdiction but is different in scope and nature. In exclusive jurisdiction, these are situations in which the doctrine of *concurrent primary jurisdiction* does not apply because the law itself clarifies that the latter does not exist. That is, the statute itself establishes an exclusive jurisdiction. In such cases we are facing a legislative mandate. Hence, when a statute expressly confers jurisdiction on an administrative body on certain types of matters, the courts will not have the authority to determine the case in the first instance. Of course, the exclusive primary jurisdiction does not preclude further judicial review of the body’s decision. (Quotes omitted and emphasis in the original.)⁵⁸

58 *SLG Semidey Vázquez v. ASIFAL*, 177 D.P.R. 657, 677 (2009).

[27] In observing the express, broad and detailed manner in which the legislator explained himself, we determine *933 that, with the exception of the remedies authorized by sec. 269 (*injunctions*), Art. 3 of the Antitrust Act, *supra*, confers exclusive primary jurisdiction on the D.A.Co. in the elucidation of any violation of subsection (a) of the article itself, that is, the use of unfair methods of competition and unfair or deceptive practices or acts in business or trade.

With regard to the determination of the court of instance in the sense that “a private entity such as the plaintiff may file a cause of action under Article 7 of the Gasoline Control Act,” from what we have already explained, we determine that it is clearly erroneous. Art. 7 of the Gasoline Act, *supra*, provides for a cause of action against the retailers and not in favor of them. On the other hand, in addition to the fact that the interpretation of such statute by the primary court was not correct, the truth is that the appellees never claimed a cause of action under Art. 7 of the Gasoline Act,

rather under Art. 7 of the Antitrust Act, *supra*.⁵⁹

59 See the claim filed by the appellees. Appendix of the *Cteriorari Petition*, pp. 79–84.

[28] However, in relation to the fact that the cause of action of the appellees can be compensated as a simple or ordinary remedy for damages and losses under Art. 1802 of the Civil Code, 31 L.P.R.A. sec. 5141, we do not find anything in the law that prevents it; on the contrary. As the Solicitor General points out in her special appearance, the following arises from the discussion outlined in the legislative history in relation to the house reports that originated the Antitrust Act:

With regard to the actions of individuals, for treble damage, these are not authorized with respect to the acts prohibited by Article 3, *without this being used to affect the right to file an ordinary action for damages and losses, if it was appropriate* *934 *in accordance with the Civil Code or other principles of law.* (Emphasis added.)⁶⁰

60 Diary of the sessions of the House of Representatives, May 22, 1964, p. 1708.

As we can see, the legislator's expressions were clear. The Legislative Assembly did not intend that a cause of action such as that in the case in question would be barred from being compensated as a simple or ordinary remedy for damages and losses under Art. 1802 of the Civil Code, *supra*. The opposite is true.

Finally, at the end of their arguments the appellees state that the cause of action in this litigation includes events occurring from 2000 to 2005, a period of time in which Art. 8 of the Gasoline Act, *supra*, had not been amended to include Art. 4A. Therefore, the appellees conclude that as such an amendment occurred after the filing of their cause of action, this—the 2005 amendment—would have a retroactive effect in violation of Art. 3 of our Civil Code.⁶¹ They are incorrect.

61 Art. 3 of the Civil Code establishes the following:

“The laws will have no retroactive effect, if they do not provide otherwise. Under no circumstances may the retroactive effect of a law prejudice the rights acquired under previous legislation.” 31 L.P.R.A. sec. 3.

First, we have already determined, based on the analysis of the amended article and the legislative history of Act No. 157, *supra*, that the intention of the Legislative Assembly was that Art. 4A of the Gasoline Act, *supra*, was to be included under the jurisdictional scope of Art. 8 of the law itself, from the amendment of 1996. This is how we interpret it.

But, in addition, if we accept the proposal of the appellees in the sense that Art. 8 of the Gasoline Act, *supra*, did not include in its jurisdictional scope Art. 4A for the year in which they filed their cause of action, we would still have to resolve that the 2005 amendment would not constitute an amendment with a retroactive effect. By this we mean the following. *935

The inclusion of Art. 4A of the Gasoline Act, *supra*, to the jurisdictional scope of Art. 8 of the Gasoline Act, *supra*, by amendment of 2005, constituted a change in the jurisdictional aspect of this article. That is, since the 2005 amendment, all causes of action under Art. 4A of the Gasoline Act, *supra*, fall under the jurisdictional scope of Art. 3 of the Antitrust Act, *supra* (10 L.P.R.A. sec. 259). As such, the 2005 amendment, in effect, resulted in the fact that private causes of action could no longer be exercised under this article and that it is the D.A.Co. that was granted exclusive primary jurisdiction.

Since 1905, this Court resolved, interpreting Art. 3 of the Civil Code, *supra*, that

... it is a principle of Spanish law, that *the statutes that regulate jurisdiction and procedure are in the public interest, and begin to govern retroactively*, [or] rather, that they are not considered retroactive in this sense, when they are included under the restrictions of this article. (Emphasis added.)⁶²

62 *American Railroad Co. of P.R. v. Hernandez*, 8 D.P.R. 516, 520 (1905).

Also in *Texas Co. v. Sancho Bonet, Tes.*, 52 D.P.R. 658, 667(1938), we note that

...as stated in 59 Corpus Juris 1173, summarizing the jurisprudence, “... The general rule that the statutes shall only be interpreted prospectively and not retrospectively or retroactively, *ordinarily is not applicable to statutes affecting the remedy or procedure, or, in other words, that general rule is subject to an exception when it is a statute relating to the remedy or procedure.*” (Emphasis added.)

Thus, considering that the amendment produced by Act No. 74-2005 to Art. 8 of the Gasoline Act, *supra*, clearly had an effect on the processes and jurisdiction of the court of instance in causes of action under Art. 4A of the Gasoline Act, *supra*, such *936 amendment is retroactive in the context that Art. 3 of the Civil Code, *supra*, prohibits it.⁶³

63 *J.R.T. v. A.E.E.*, 133 D.P.R. 1, 13, (1993). See also: *Texas Co. v. Sancho Bonet, Tes.*, 52 D.P.R. 658, 667 (1938); R.E. Bernier and J.A. Cuevas Segarra, *Aprobación e interpretación de las leyes en Puerto Rico [Approval and interpretation of the laws in Puerto Rico]*, 2nd ed., San Juan, Pubs. J.T.S., 1987, Vol. I, Chap. 63.

IV

Due to all of the above, *the Resolution issued by the Court of Appeals is revoked and, consequently, the cause of action based on Art. 4A of the Gasoline Act, supra, before the Court of First Instance, is dismissed, due to lack of jurisdiction.*

A judgment in this sense will be issued.

The Presiding Judge Mr. Hernández Denton issued a dissenting opinion, which was joined by the Associate Judge Ms. Rodríguez Rodríguez.

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Dissenting opinion issued by the Presiding Judge Mr. Hernández Denton, which was joined by the Associate Judge Ms. Rodríguez Rodríguez.

Given that neither the Gasoline Producers, Refiners and Distributors-Wholesalers Control Act, Act No. 3 of March 21, 1978, as amended, 23 L.P.R.A. sec. 1101 *et seq.* (Gasoline Act), nor the Antitrust and Restriction of Trade Act, Act No. 77 of June 25, 1964, as amended, 10 L.P.R.A. sec. 257 *et seq.* (Antitrust Act), prevent a market participant from filing an action for damages for violations of the prohibitions of the aforementioned laws, we dissent from the course followed by a majority of this Court. In essence, we cannot *937 agree because the outcome of the Court's Opinion is contrary to the purposes of the antitrust statutes in dispute.

I

The plaintiffs, Aguadilla Paint Center, Inc., CB Gasoline Service Group, and Micada de Puerto Rico, Inc. are retailers or retail sellers of gasoline imported, distributed, and sold by Esso Standard Oil (Esso), in other words, they are brand retailers. In 2004, the retailers filed a claim against Esso in which they charged it with incurring in the unlawful practice of operational association and discrimination of prices and discounts, in contravention of Art. 4A of the Gasoline Act, 23 L.P.R.A. sec. 1104a, and Art. 7 of the Antitrust Act, 10 L.P.R.A. sec. 263. Likewise, they requested compensation under Art. 1802 of the Civil Code of Puerto Rico, 31 L.P.R.A. sec. 5141. Specifically, the plaintiffs allege that Esso controls the operation of gasoline stations in violation of Art. 4A, because it establishes the cost of the gasoline, the amount of gasoline sold, the frequency with which the gasoline is purchased, the delivery thereof, the method of payment, the suggested sale price and the rental of the service stations. On the other hand, the cause of action for violation of Art. 7 of the Antitrust Act is based on the fact that, allegedly, Esso maintains a price discrimination system in relation to equally situated buyers.

For its part, Esso requested the dismissal alleging, in essence, that these two causes of action can only be filed by the Department of Justice. Therefore, the plaintiffs lacked active standing.

The Court of First Instance ruled that the motion for dismissal was “inadmissible” and the intermediate appeal venue maintained that opinion by not issuing the requested order, because it was not in a position to intervene in that procedural stage. *938 Dissatisfied, Esso approached this court. For their part, the plaintiffs, whose arguments were excluded from the Opinion of the Court, argue that they are aware that there are a series of remedies in the aforementioned statutes that have been entrusted to the Department of Justice. Specifically, they argue that the preliminary and permanent *injunctions* recognized in such laws to paralyze prohibited actions are the exclusive competence of the Secretary of Justice and that is who has active standing to seek them. However, the plaintiffs do not seek to interfere with those remedies. Their only request is to be allowed to demonstrate that Esso caused them damages by violating the precepts of both laws by incurring in operational association practices and price discrimination. Even though they originally requested it in their claim, the plaintiffs argue before us that they do not claim the treble damages remedy, recognized in the antitrust statutes of the United States and Puerto Rico. Again, they only want to be allowed to demonstrate how, by engaging in the practices prohibited by the aforementioned statutes, damage was caused to them that, in some cases, resulted in the bankruptcy of family businesses.

II

A. Antitrust statutes

The Gasoline Act aims to combat the control of energy product supplies and eliminate the competitive advantages existing in the gasoline market. Therefore, our Legislative Assembly sought competitive uniformity and implemented prohibitions on discriminatory and price control practices. A few months later, Act No. 73 of June 23, 1978 (23 L.P.R.A. sec. 1131 *et seq.*) (Law 73) was approved, for the purpose of declaring the gasoline industry—in all its facets—as having the status of being in the *public interest*. *939

In turn, Law 73, *supra*, implemented a series of prohibitions on lease and sublease agreements, and other types of association between wholesalers and retailers. In addition, it established that wholesalers are required to submit to the Department of Justice's Antitrust Affairs Office (O.A.M.) a copy of all their contracts with retailers. Likewise, Law 73 provided that it recognized the existence of the Antitrust Act and that nothing in the former would contravene the latter. That is, the Legislative Assembly was aware of the Antitrust Act and its operation.

In harmony with the above, Art. 4A of the Gasoline Act, 23 L.P.R.A. sec. 1104a, provides that

no... distributor-wholesaler may, by agreement... directly operate a gasoline retail service station in a manner that prevents its complete operational separation. No... distributor-wholesaler may, by agreement ... directly impose, require, fix or limit the profit margin and/or retail price of gasoline... at the retail service station.

Likewise, Art. 8 of the Gasoline Act, 23 L.P.R.A. sec. 1108, provides that “any violation of sec. 1102, 1102a, 1104, 1104a, 1105 and 1105a, of this title shall constitute an unfair or deceptive practice or act and shall be subject to the provisions of sec. 257 *et seq.*, of Title 10.” (Emphasis added.) The latter on sec. 257 *et seq.* of Title 10 refers to the Antitrust Act.

On the other hand, the Antitrust Affairs Office of the Department of Justice has a series of remedies and powers to monitor compliance with the Antitrust Act. It should be noted that all sections of that law referred to by the Opinion of this Court provide that the O.A.M. “may” perform certain acts. *940

On the other hand, Art. 7 of the Antitrust Act provides, where relevant, that:

(a) It shall be illegal for any person, directly or indirectly, to discriminate in price between different buyers of commercial things of the same grade and quality, when those things are sold for use, consumption or resale in Puerto Rico, and where the effect of such discrimination may be to substantially reduce competition or tend to create a monopoly on any line of commerce in Puerto Rico or affect, destroy or prevent competition with any person who has granted or knowingly received the benefit of such discrimination, or any customer of one of these. 10 L.P.R.A. sec. 263.

Finally, Art. 12 of the Antitrust Act provides:

(a) Any person who is injured in their business or property by another person, due to acts, or attempted acts, prohibited or declared illegal by the provisions of this chapter, except for those of Sections 259 and 261 of this title, may sue because of such acts before the Court of First Instance and shall be entitled to

recover three (3) times the amount of the damages and losses experienced, plus the costs of the proceeding and a reasonable amount for attorney's fees. (Emphasis added.) 10 L.P.R.A. sec. 268.

As this last article excludes the actions under 10 L.P.R.A. sec. 259, today this Venue concludes that not just any person may file that action, rather the O.A.M. will be the only body entitled in accordance with that provision of the Antitrust Act. Therefore, it orders the dismissal of the causes of action under the Gasoline Act and the Antitrust Act, although it allows the case to continue with regard to the allegations of damages according to Art. 1802 of the Civil Code, *supra*.

As we will see later, that analysis is contrary to what is expressed in *Pressure Vessels P.R. v. Empire Gas P.R.*, 137 D.P.R. 497 (1994). There, we established the origin and encompassing purposes of Art. 2 of the Antitrust Act, *941 10 L.P.R.A. sec. 258. It seeks to avoid any act, contract or conspiracy to unreasonably restrain business or trade in Puerto Rico. To this end, our Legislative Assembly adopted the same text as Sec. 1 of the *Sherman Act*, 15 U.S.C.A. sec. 1. See: *Pressure Vessels*, *supra*; *Northern Pac. R. Co. v. United States*, 356 U.S. 1, 4 (1958).

In addition, we established that both the text of our article and federal doctrine agree that there are three requirements that must be established to demonstrate a violation of this provision: (1) there must be a contract, combination or conspiracy between two or more entities, (2) which unreasonably restricts business or trade (3) in Puerto Rico. *Pressure Vessels P.R. v. Empire Gas P.R.*, *supra*, p. 509. Finally, in the aforementioned case, we resolved that a plaintiff may bring evidence to demonstrate the unreasonable nature of certain contracts based on their substantially adverse effect on free competition, this as a vehicle to prove that, "as a result of the violation of the law, the plaintiff has suffered damage." *Id.*, p. 520.

B. Grant of regulatory power

On June 23, 1978, the Legislative Assembly passed two laws that concern us in relation to the current dispute. The first is Act No. 72, which had the purpose of adding to Art. 3 of the Antitrust Act, *supra*, a provision "in order to authorize the Antitrust Affairs Office to file and process complaints for violations of this article in the Department of Consumer Affairs," "to authorize said Department to resolve said complaints and impose the penalties provided herein" and "to establish the procedural process for the judicial review of said complaints." See Act No. 72 of June 23, 1978 (1978 Puerto Rico Laws 256–257). In addition, that law clearly establishes that what was desired was "to provide an *additional forum to* *942 *the existing ones* where unfair acts or methods in business and trade can be reviewed." (Emphasis added.) *Id.*, page 257.

In turn, the second approved law, Act No. 73, was established with the purpose of "authorizing the Departments of Justice, Trade and Consumer Affairs, the Planning Board, the Administration of Regulations and Permits, and the Public Service Commission to adopt and implement the regulations necessary to put into effect the purposes and objectives of this law." 1978 Laws of Puerto Rico 259. The Statement of Reasons of this law provides that "government intervention in its exercise of the State's Reasoning Power is necessary to avoid dislocation in the integrity, functionality and competition that must prevail in the gasoline market in Puerto Rico, an industry vital to the best interests of the citizens and the economic dynamics of the country." *Id.*, page 260. Later, that same law prescribes in Art. 5 that "nothing herein provided or enacted by regulation or otherwise, in accordance with the provisions of this law, shall be interpreted as creating an exemption or conferring immunity as to the applicability of the provisions of Act No. 77 of June 25, 1964, as amended." *Id.*, page 264.

This being the case, the Legislative Assembly wanted to add an administrative remedy with the purpose of adding tools to the antitrust fight. Nonetheless, nothing herein provided on the administrative powers and procedure was intended to eliminate the remedies recognized by the Antitrust Act, as well provided for in Art. 5 of Act No. 73, *supra*.

That legislative action is entirely consistent with the 1970 amendment to the Antitrust Act. By means of Act No. 67 of May 30, 1970, an amendment was made to Art. 3 of the Antitrust Act, *supra*, which provides that "without [text cut-off] *943 authorized in Article 13 [which deals with the remedy of *injunction*], the Antitrust Affairs Office, by means of rules and regulations enacted ... may proscribe specific acts or practices..." (Scholiums omitted.) Act No. 67 of May 30, 1970 (1970 Puerto Rico Laws 185).

Therefore, Art. 3 of the Antitrust Act, *supra*, as amended, simply recognizes the administrative power to regulate unlawful practices and provides a venue before the Department of Consumer Affairs to examine violations of the statutes discussed herein.

III

On the basis of the foregoing, we disagree with the result of the majority opinion. We consider that nothing in the statutes we have discussed prevents the plaintiffs' active standing and that, despite the fact that, according to Art. 12 of the Antitrust Act, *supra*, Art. 3 is excluded, the plaintiffs' action in no way contravenes the legislative delegation of power to regulate, which is granted in that Art. 3. In summary, the claimants should be allowed to prove a case of damages for violations of the specific provisions of the statutes discussed herein. As follows.

The Gasoline Act was adopted following the passage of the Antitrust Act. Likewise, the former expresses that the Legislative Assembly was aware of the provisions of the latter, at the time of approving it. The Gasoline Act states in its Art. 8 that any violation of several of its articles, including Art. 4A, "shall constitute an unfair [or] deceptive practice or act and shall be subject to the provisions of *sec. 257 et seq.* of Title 10." 23 L.P.R.A. *sec. 1108*.

When we examine *sec. 257 et seq.* of Title 10, that is, the Antitrust Act, we find that the Civil Code of Puerto Rico, *supra*, asks us: what ale [text cut-off] Art. *944 12 recognizes a cause of action to "any person who is harmed in their businesses or properties by another person, by reason of acts, or attempted acts, prohibited or declared illegal by the provisions of this chapter, except those of *sections 259 and 261...*" 10 L.P.R.A. *sec. 268*.

In harmony with the legislative purposes of our statutes and the federal statutes on which local legislation is based, *that cause of action is entirely independent of the legislative power to regulate that was conferred on the administrative agencies by Art. 3 of the Antitrust Act, supra*.

This is important because, according to the Court's Opinion, the practices that violate Art. 8 of the Gasoline Act, *supra*, would only be subject to action in accordance with the aforementioned Art. 3 of the Antitrust Act, which provides that "[t]he unfair methods of competition, as well as unfair or deceptive practices or acts in business or trade, are hereby declared illegal." 10 L.P.R.A. *sec. 259*. That is, because Art. 8 of the Gasoline Act declares "unfair and deceptive acts" to be prohibited in said law and that, as Art. 3 of the Antitrust Act is the one that prohibits "unfair and deceptive acts," and this, in turn, is excluded from Art. 12 of the Antitrust Act that recognizes the treble damages action, then the plaintiffs do not have that cause of action available for violations of the Gasoline Act.

That textual interpretation—which the petitioner proposes and which this Court fully adopts—is entirely contrary to the purposes of both laws. In addition, the items claimed by the plaintiffs should not be construed as the only items of those laws that allow an action for damages. While that interpretation made by the Opinion of this Court allows the plaintiffs to continue with their cause of action in damages in accordance with Art. 1802 of the Code *945 [text cut-off] -gations could be filed by the plaintiffs to demonstrate that it was the prohibited practices of the defendant that caused the damages they seek to indemnify, if the causes of action under the Antitrust Act and the Gasoline Act have been dismissed?

We believe that a different interpretation could have been reached. Even more so in light of the fact that the Court's own Opinion recognizes the direct origin of our antitrust statutes from their federal analogs.

In the United States, private antitrust actions play a critical role in the antitrust regime. *Antitrust Laws and Trade Regulation*, Chap. 160, Sec. 160.01, page 160-2 (Mathew Reader). "In enacting the antitrust laws, the US Congress was convinced that 'private antitrust litigation is one of the surest weapons for effective antitrust enforcement' and that private suits are an important element of the Nation's antitrust enforcement' scheme The Supreme Court has labeled 'the private action as a vital means of enforcing the antitrust policy of the United States' and as an 'important weapon of antitrust enforcement.' " (Emphasis added and scholiums omitted.) Id.

Therefore, our interpretation is that the alleged unlawful action of the respondents could be prohibited by the Antitrust Act and the plaintiffs thus could prove it in accordance with our opinion in *Pressure Vessels P.R. v. Empire Gas P.R.*, *supra*, which

essentially allows a market participant to claim damages for a prohibited act under the antitrust statute.

In fact, under federal law, the treble damages remedy is available in Sec. 4 of the *Clayton Act*. The text of that section is extremely simple and simply recognizes a cause of action to any person affected by any act prohibited by an antitrust law. 15 U.S.C.A. sec. 15. To obtain that remedy, the plaintiff only has to prove that the damage was caused by an [text cut-off] acCivil [sic] of Puerto Rico, *supra*, we can ask ourselves: what allegation [text cut-off] *946 antimonopoly. *Antitrust Laws and Trade Regulation* Chap. 160, Sec. 161.02, p. 161-9 (Mathew Bender) (“Thus, in antitrust cases, it is not sufficient simply to demonstrate an injury, even injury directly caused ‘by the defendant. The injury will not be considered antitrust injury unless it was caused by reason of a violation of the antitrust laws’”). This demonstrates that Congress wanted to allow market participants to file a damage action for the mere fact of another person having performed an antitrust practice.

Therefore, we would have concluded that the claim filed by the appellants could continue. With its decision, this Court puts on hold an important element of the antitrust fight, in particular regarding the fuel distribution business in Puerto Rico. It is sufficient to look back over decades to remember the undesirable practices we experienced in the Country during the oil crisis of the 1970s. Today, turning our back on the public policy of the United States, the base on which we formulate our statutes related to this matter, this Court binds the hands and closes the doors of justice to those who claim to have suffered damages that will now be more difficult to demonstrate.

From the federal regulations described above, it is evident that, since the beginning of the last century, legislation provided that competitors would play a central role in the fight against the illicit control of markets, by submitting actions to recover the damage caused by monopolistic actions. We do not see why Puerto Rican gasoline market participants cannot do the same, when our legislation is also intended to empower those affected with standing to claim remedies for unfair practices.

IV

As an epilogue, this Court’s decision is issued *947 at a time when the Antitrust Affairs Office’ ability to advance the legislative purposes is in question. From the information published by the Department of Justice regarding its budget for the current fiscal year, it arises that, since 2009, the Office has lost almost forty percent of its staff, it currently has only four attorneys, and, during the same period, it has had fifteen percent of its budget cut. See Department of Justice, *Approved Budget for 2012*, available at: [http:// www.pr.gov/presupuesto/aprobado—2012/justicia.htm](http://www.pr.gov/presupuesto/aprobado—2012/justicia.htm).

In sum, we would have allowed the litigation to continue on its merits rather than settle it summarily, without allowing the plaintiff to establish its case. The dismissal of private actions against the illicit control of markets in our jurisdiction is contrary to both the intent and spirit of the Antitrust Act and the Gasoline Act, and deprives gasoline retailers of a very important instrument to vindicate their rights in response to unfair wholesaler practices.